COMP3204/COMP6223: Computer Vision

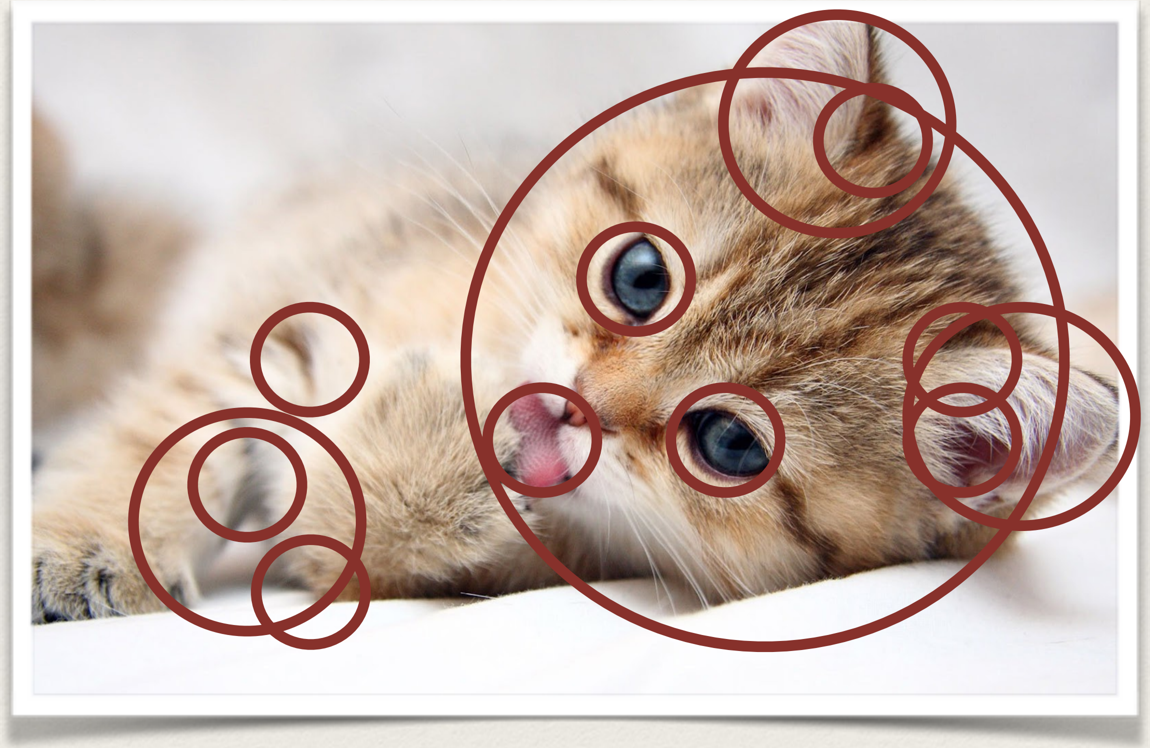
Local interest points

Jonathon Hare
jsh2@ecs.soton.ac.uk

- Finding stable (repeatable) interest points is a key problem in modern computer vision
  - Applications in areas such as tracking, matching, image alignment, making robust features for classification and search, robot navigation, 3d reconstruction, …
  - We'll look at some of these in more detail in future lectures

What makes a good interest point?

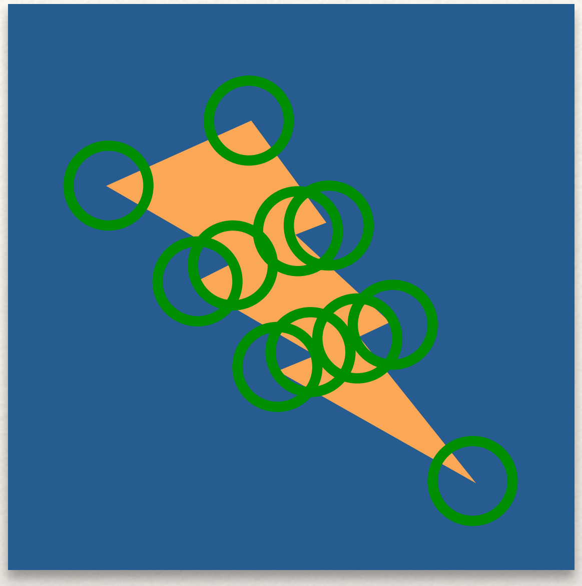
- Invariance to brightness change (local changes as well as global ones)
- Sufficient texture variation in the local neighbourhood
  - But not too much!
- Invariance to changes between the angle/position of the scene to the camera



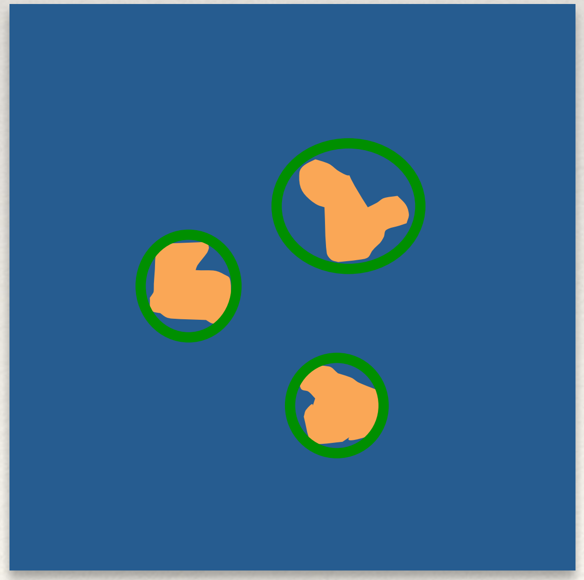
Demo: Stable local interest points

So, how do we find them?

- Lots of different types of interest point types to choose from.
    - We'll focus on two specific types and look in detail at common detection algorithms:
        - Corner detection - *Harris and Stephens*
        - Blob Detection - *Difference-of-Gaussian Extrema*

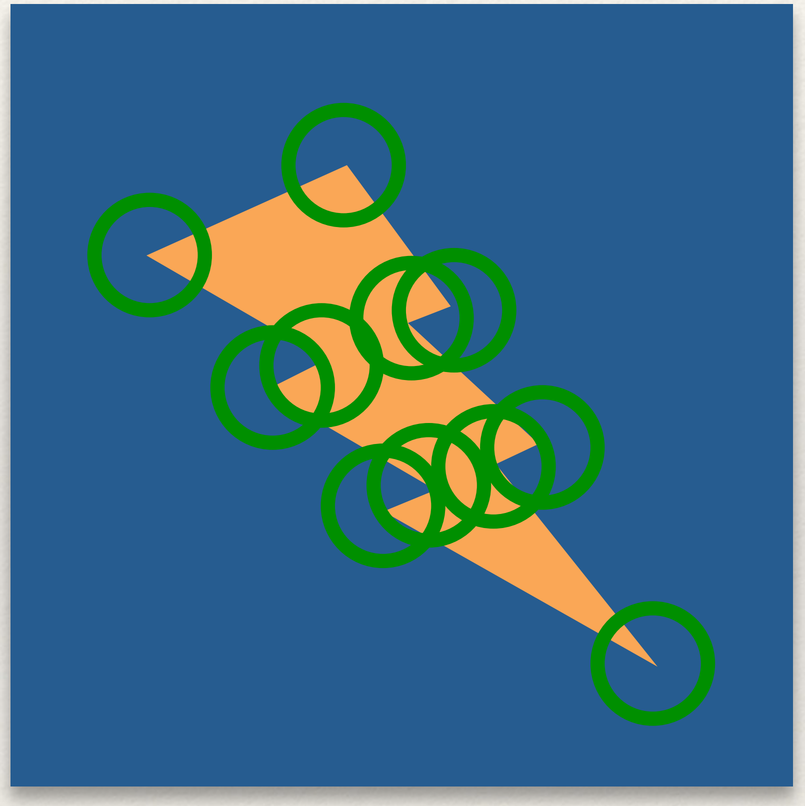


corners



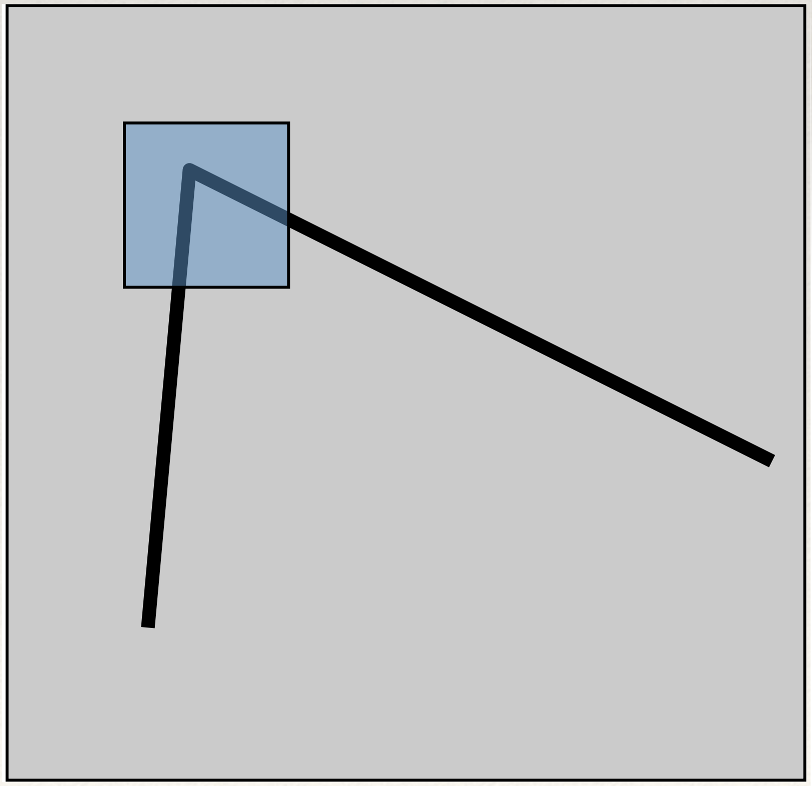
blobs

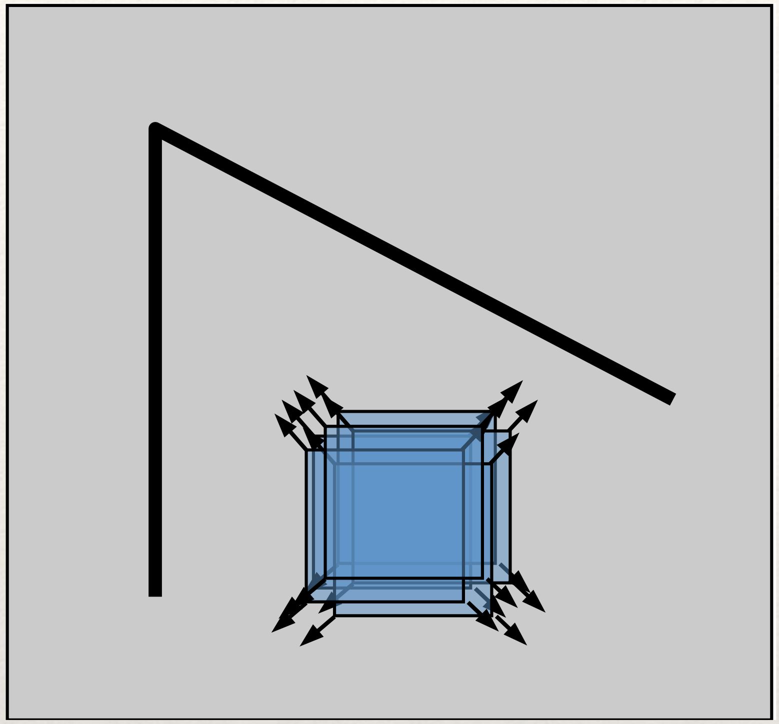
The Harris and Stephens corner detector



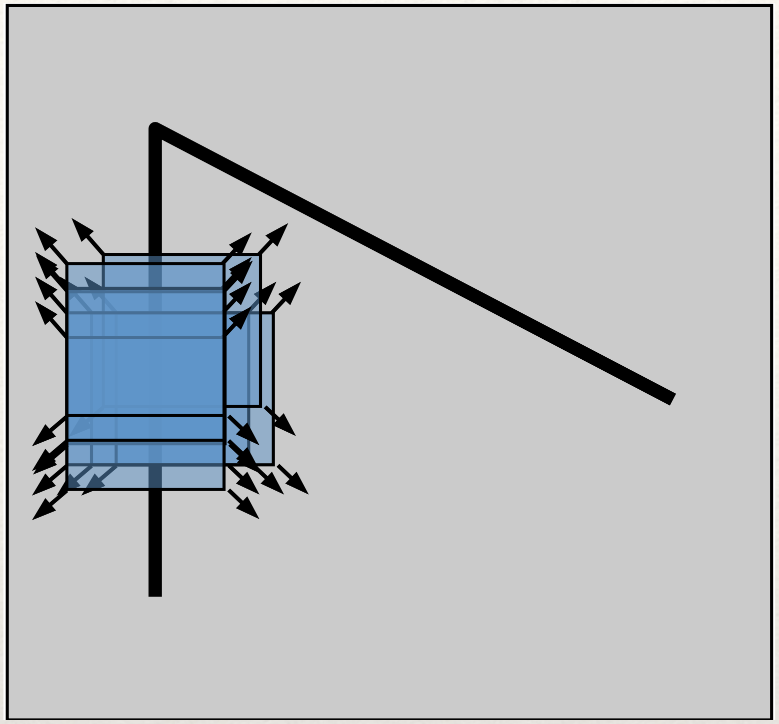
Basic Idea

- Search for corners by looking through a small window.
- Shifting that window by a small amount in *any direction* should give a *large change* in intensity

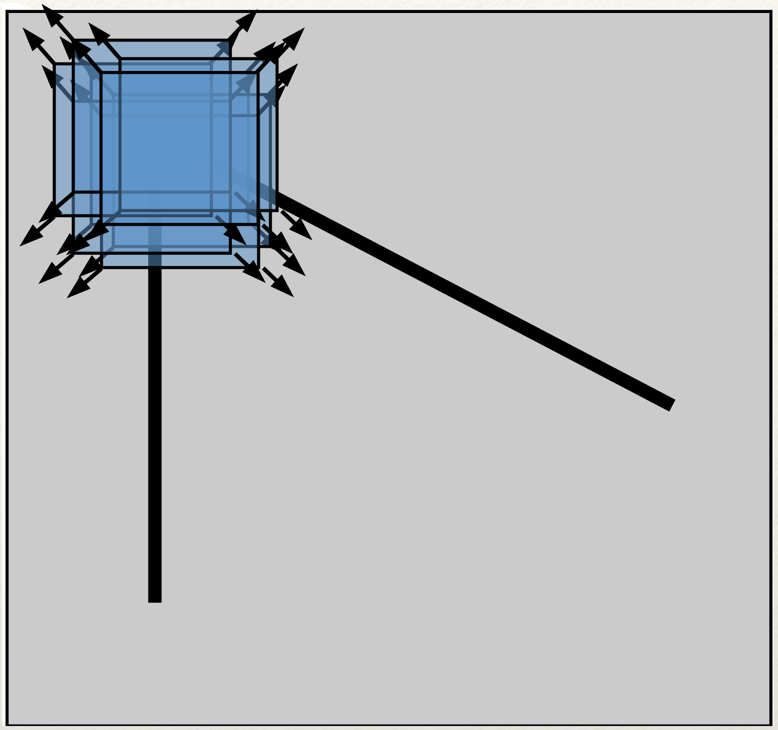




"flat" region: no change in all directions



"edge": no change along the edge direction



"corner": significant change in all directions

Harris & Stephens: Mathematics

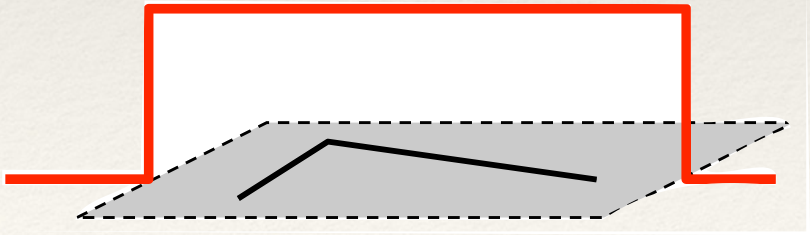
Weighted average change in intensity between a window and a shifted version [by (**Δ**x,**Δ**y)] of that window:

$$E(x,y) = \sum_{W} f(x_i,y_i)[I(x_i,y_i) - I(x_i+\Delta x, y_i+\Delta y)]^2$$

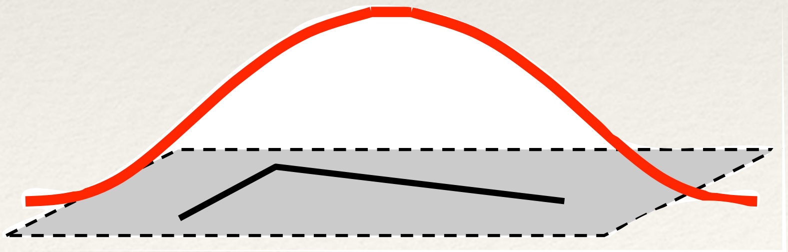
weighting function

intensity in window

intensity in shifted window



flat



Gaussian

Harris & Stephens: Mathematics

- The Taylor expansion allows us to approximate the shifted intensity.
- Taking the first order terms we get this:

$$I(x_i + \Delta x, y_i + \Delta y) \approx I(x_i, y_i) + \begin{bmatrix} I_x(x_i, y_i) & I_y(x_i, y_i) \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

(partial derivatives of the image)

Harris & Stephens: Mathematics

- Substituting and simplifying gives:

$$
\begin{aligned}
E(x,y) &= \sum_{W}[I(x_i,y_i) - I(x_i+\Delta x, y_i+\Delta y)]^2 \\
&= \sum_{W}\left(I(x_i,y_i) - I(x_i,y_i) - [I_x(x_i,y_i) \quad I_y(x_i,y_i)]\begin{bmatrix}\Delta x\\ \Delta y\end{bmatrix}\right)^2 \\
&= \sum_{W}\left(-[I_x(x_i,y_i) \quad I_y(x_i,y_i)]\begin{bmatrix}\Delta x\\ \Delta y\end{bmatrix}\right)^2 \\
&= \sum_{W}\left([I_x(x_i,y_i) \quad I_y(x_i,y_i)]\begin{bmatrix}\Delta x\\ \Delta y\end{bmatrix}\right)^2 \\
&= [\Delta x \quad \Delta y]\begin{bmatrix}\sum_{w}\left(I_x(x_i,y_i)\right)^2 & \sum_{w} I_x(x_i,y_i)I_y(x_i,y_i) \\ \sum_{w} I_x(x_i,y_i)I_y(x_i,y_i) & \sum_{w}\left(I_y(x_i,y_i)\right)^2\end{bmatrix}\begin{bmatrix}\Delta x\\ \Delta y\end{bmatrix} \\
&= [\Delta x \quad \Delta y]\mathbf{M}\begin{bmatrix}\Delta x\\ \Delta y\end{bmatrix}
\end{aligned}
$$

Structure Tensor

The **square symmetric** matrix **M** is called the *Structure Tensor* or the *Second Moment matrix*

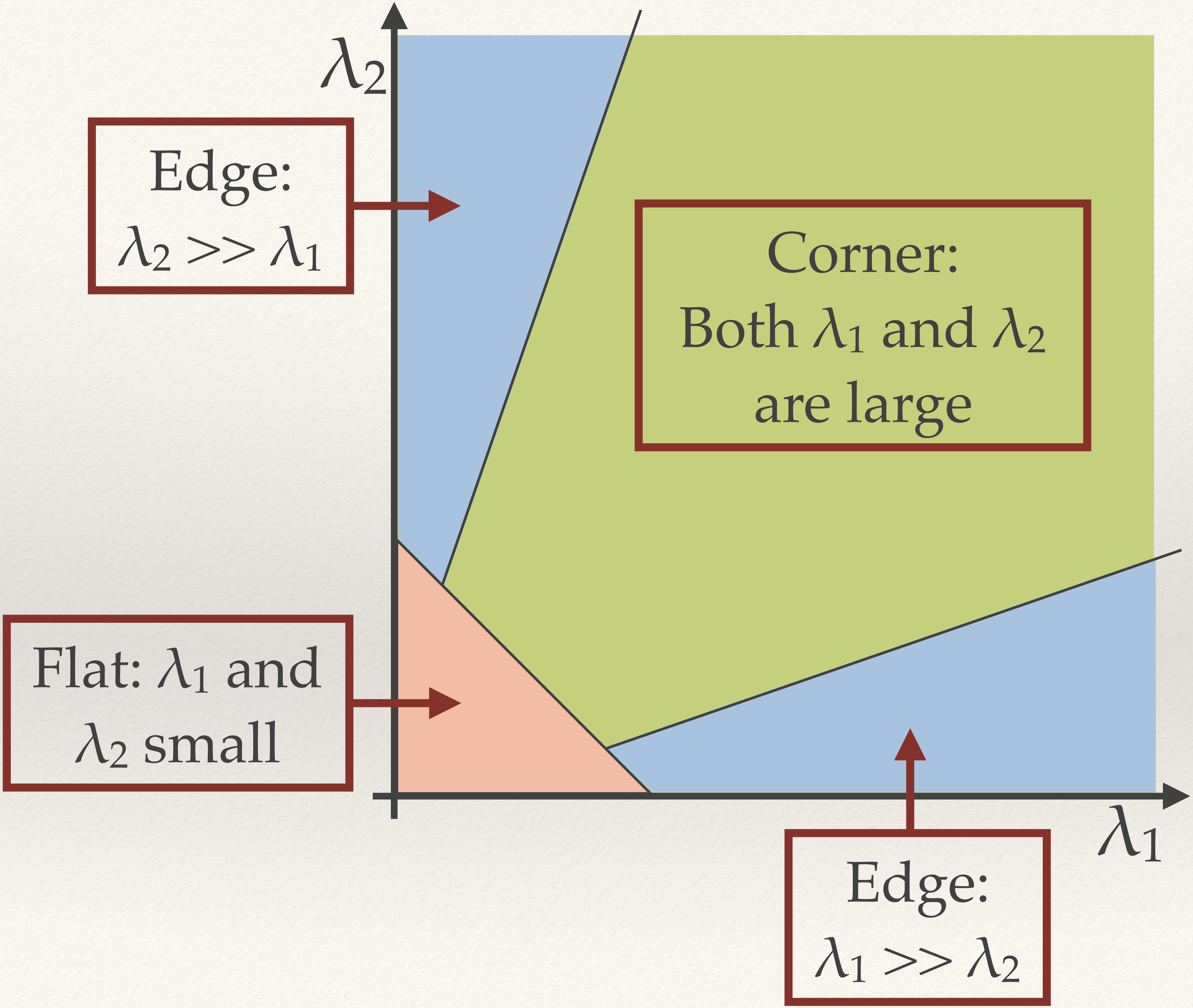
$$\mathrm{M} = \begin{bmatrix} \sum_w \left(I_x(x_i, y_i)\right)^2 & \sum_w I_x(x_i, y_i) I_y(x_i, y_i) \\ \sum_w I_x(x_i, y_i) I_y(x_i, y_i) & \sum_w \left(I_y(x_i, y_i)\right)^2 \end{bmatrix}$$

It concisely encodes the how the local shape intensity function of the window changes with small shifts

Eigenvalues of the Structure Tensor

- Think back to Lecture 3 where we looked at covariance matrices…
    - As with the 2d covariance matrix, the structure tensor describes an ellipse: $x^{T}\mathbf{M}x=c$ (this is a *quadratic form*)
    - The eigenvalues and vectors tell us the rates of change and their respective directions

Demo: Structure Tensor eigenvalues



Edge:
$\lambda_2 >> \lambda_1$

Corner:
Both λ_1 and λ_2
are large

Flat: λ_1 and
λ_2 small

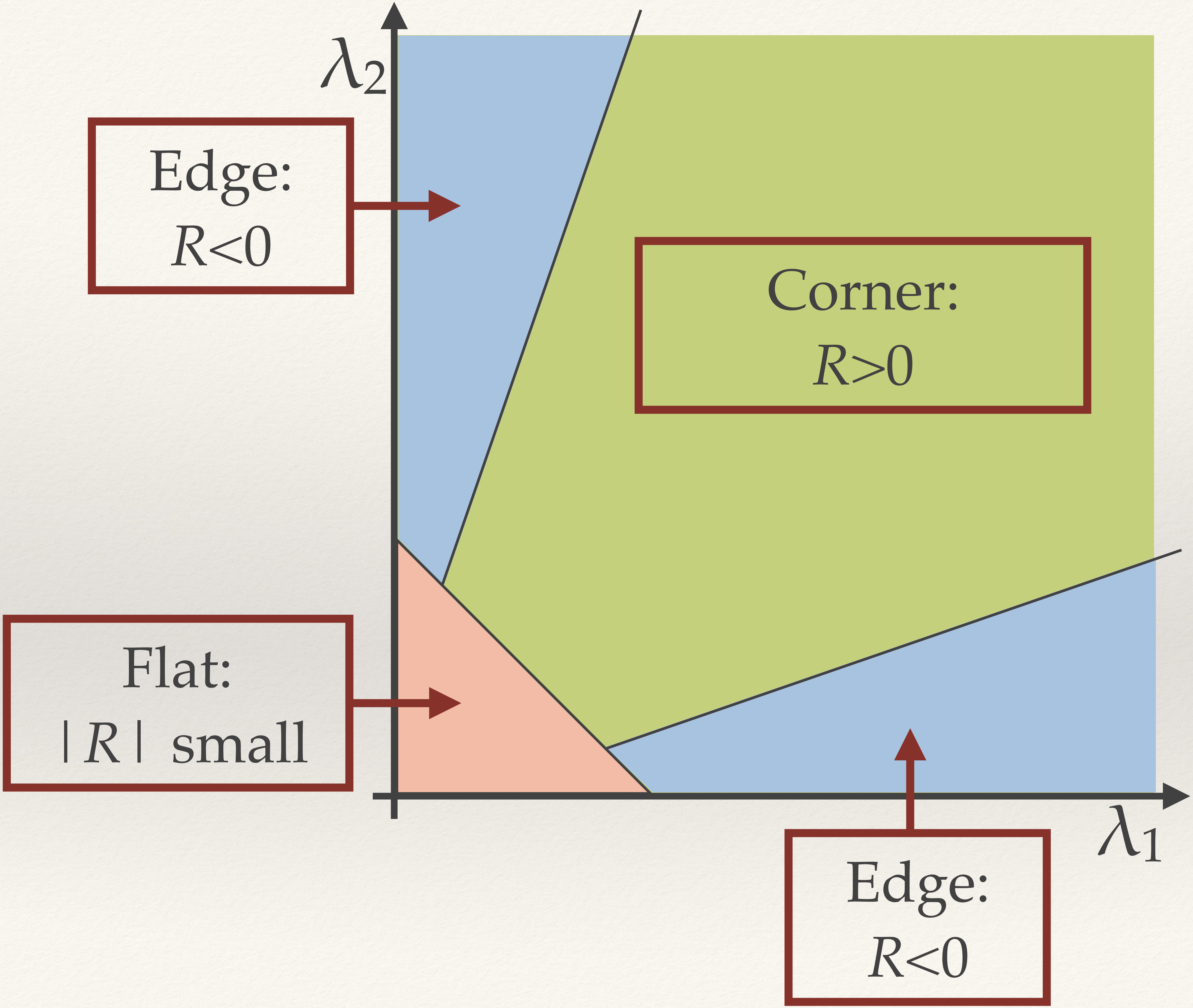
Edge:
$\lambda_1 >> \lambda_2$

Harris & Stephens Response Function

- Rather than compute the eigenvalues directly, Harris and Stephens defined a corner response function in terms of the determinant and trace of **M**:

$$\det(\mathbf{M}) = M_{00}M_{11} - M_{01}M_{10} = M_{00}M_{11} - M_{10}^{2} = \lambda_1\lambda_2$$
$$\operatorname{trace}(\mathbf{M}) = M_{00} + M_{11} = \lambda_1 + \lambda_2$$
$$R = \det(\mathbf{M}) - k\,\operatorname{trace}(\mathbf{M})^2$$

k is a small empirically set constant (usually 0.04 - 0.06)



Edge:
$R<0$

Corner:
$R>0$

Flat:
$|R|$ small

Edge:
$R<0$

Demo: Harris & Stephens Response

Harris & Stephens Detector

- Simple algorithm:
  - Take all points with the response value above a threshold
  - Keep only the points that are local maxima (i.e. where the current response is bigger than the 8 neighbouring pixels)

Demo: Thresholded Harris & Stephens Response

Demo: Thresholded Harris & Stephens Points

Scale in Computer Vision

The problem of scale

- As an object moves closer to the camera it get larger with more detail… as it moves further away it gets smaller and loses detail…
- If you're using a technique that uses a fixed size processing window (e.g. Harris corners, or indeed anything that involves a fixed kernel) then this is a bit of a problem!

Scale space theory

- Scale space theory is a formal framework for handling the scale problem.
  - Represents the image by a series of increasingly smoothed/blurred images parameterised by a scale parameter t.
  - t represents the amount of smoothing.
  - **Key notion:** Image structures smaller than sqrt(t) have been smoothed away at scale t.

The Gaussian Scale Space

- Many possible types of scale space are possible (depending on the smoothing function), but only the Gaussian function has the desired properties for image representation.
    - These provable properties are called the "*scale space axioms*".

Gaussian Scale Space

Formally, Gaussian scale space defined as:

$$L(\cdot,\cdot;t) = g(\cdot,\cdot;t) * f(\cdot,\cdot)$$

Note: convolution is over *x*, *y* for fixed *t*

where $t \geq 0$ and,

$$g(x,y;t) = \frac{1}{2\pi t} e^{-(x^2+y^2)/2t}$$

Note: $t = \sigma^2$ = variance of the Gaussian

Normally, only a fixed set of values of *t* are used - it's common to use integer powers of 2 or sqrt(2)

$t=0$

$t=1$

$t=2$

$t=4$

$t=16$

$t=32$

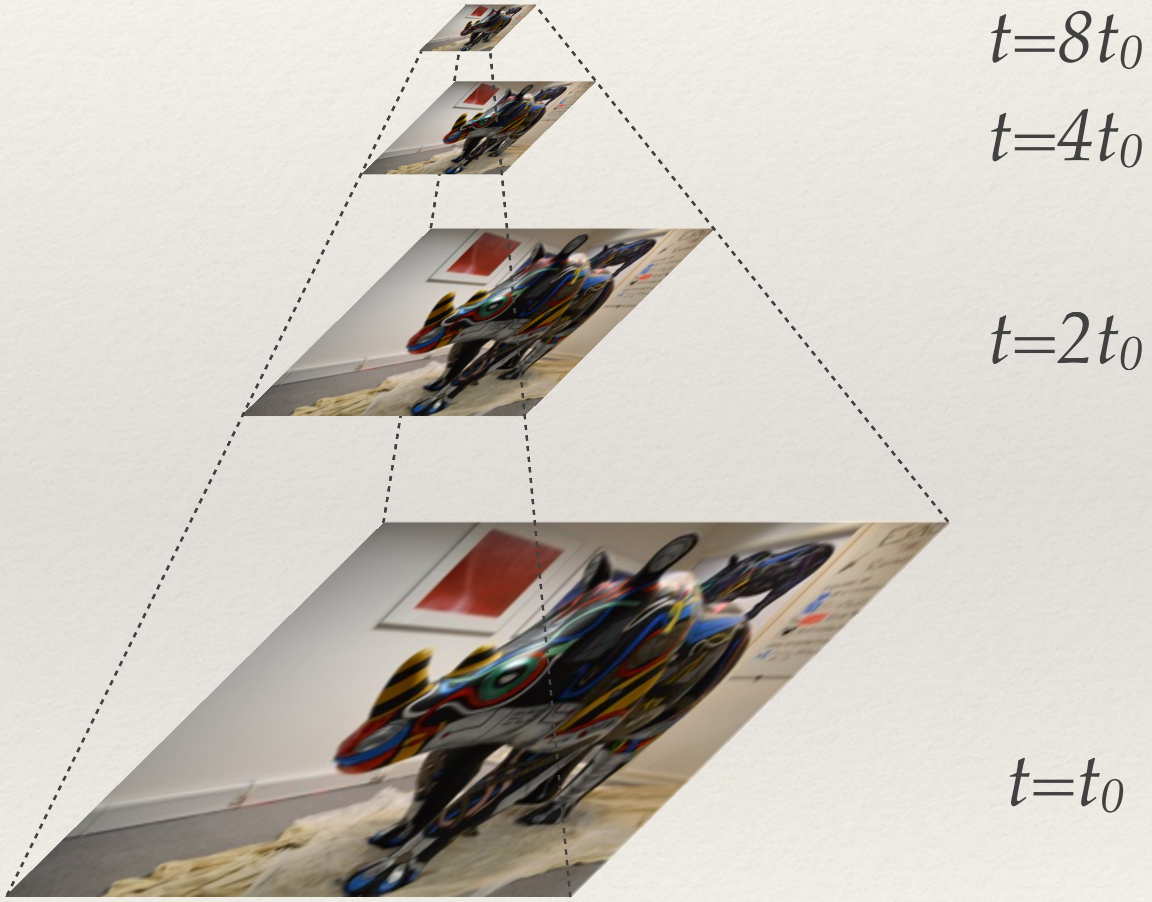
Nyquist-Shannon Sampling theorem

If a function x(t) contains no frequencies higher than B hertz, it is completely determined by giving its ordinates at a series of points spaced 1/(2B) seconds apart.

…so, if you filter the signal with a low-pass filter that halves the frequency content, you can also half the sampling rate without loss of information…

Gaussian Pyramid

- Every time you double *t* in scale space, you can half the image size without loss of information!
  - Leads to a much more efficient representation
    - faster processing
    - less memory



Multi-scale Harris & Stephens

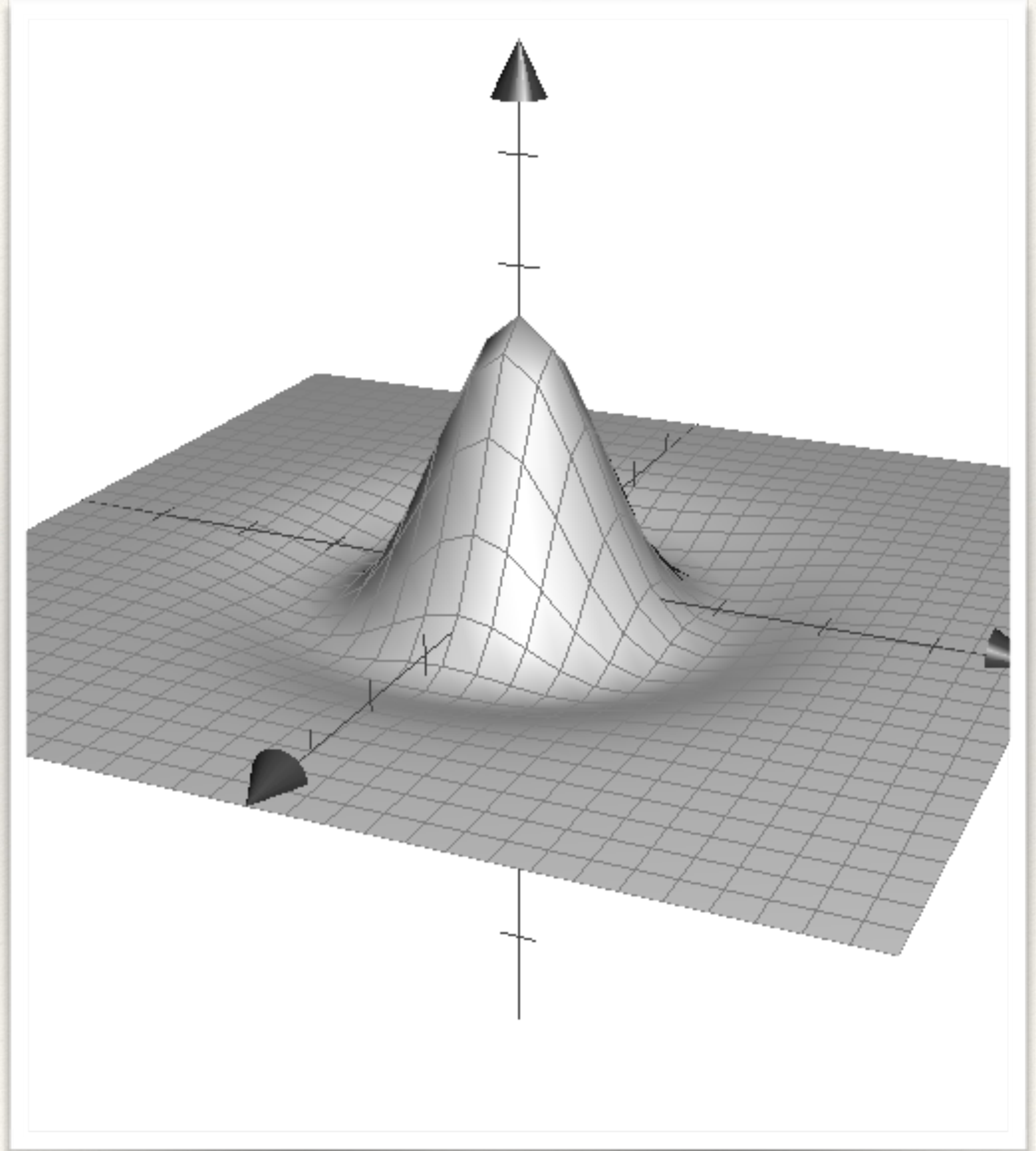
- Extending the Harris and Stephens detector to work across scales is easy…
  - We define a Gaussian scale space with a fixed set of scales and compute the corner response function at every pixel of each scale and keep only those with a response above a certain threshold.

Demo: Multi-scale Harris & Stephens

Blob Detection

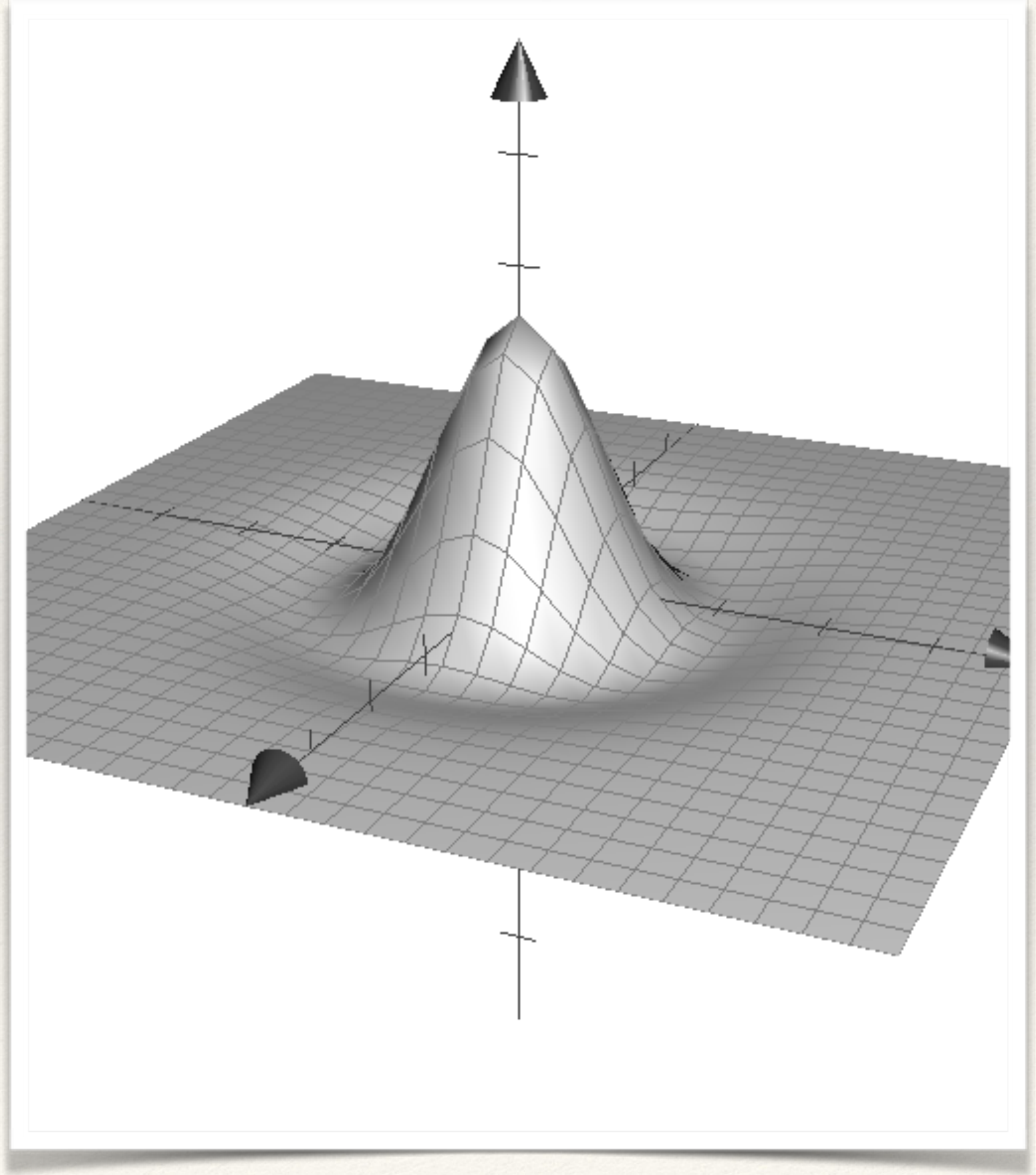
Recap: Laplacian of Gaussian

- Recall that the LoG is the second derivative of a Gaussian
    - Used in the Marr-Hildreth edge detector
        - Zero crossings of LoG convolution



Laplacian of Gaussian

- By finding local minima or maxima, you get a blob detector!

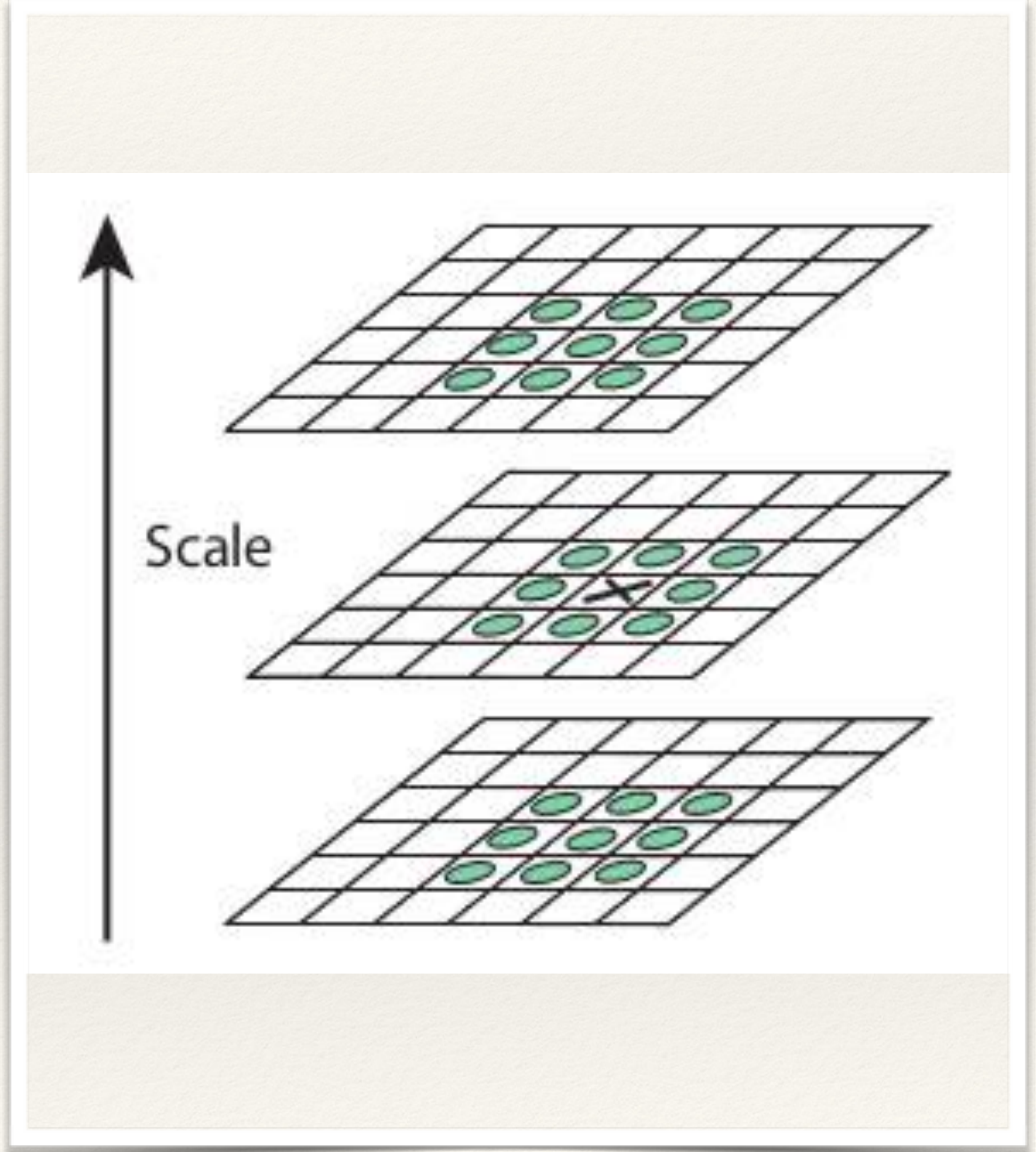


Scale space LoG

- Normalised scale space LoG defined as:

$$\nabla^2_{norm} L(x, y; t) = t(L_{xx} + L_{yy})$$

- By finding extrema of this function in scale space, you can find *blobs* at their representative scale (~sqrt(2t))
  - Just need to look at the neighbouring pixels!



Very useful property: *if a blob is detected at* $(x_0, y_0; t_0)$ *in an image, then under a scaling of that image by a factor* s*, the same blob would be detected at* $(sx_0, sy_0; s^2t_0)$ *in the scaled image.*

Scale space DoG

- In practice it’s computationally expensive to build a LoG scale space.
- But, the following approximation can be made:

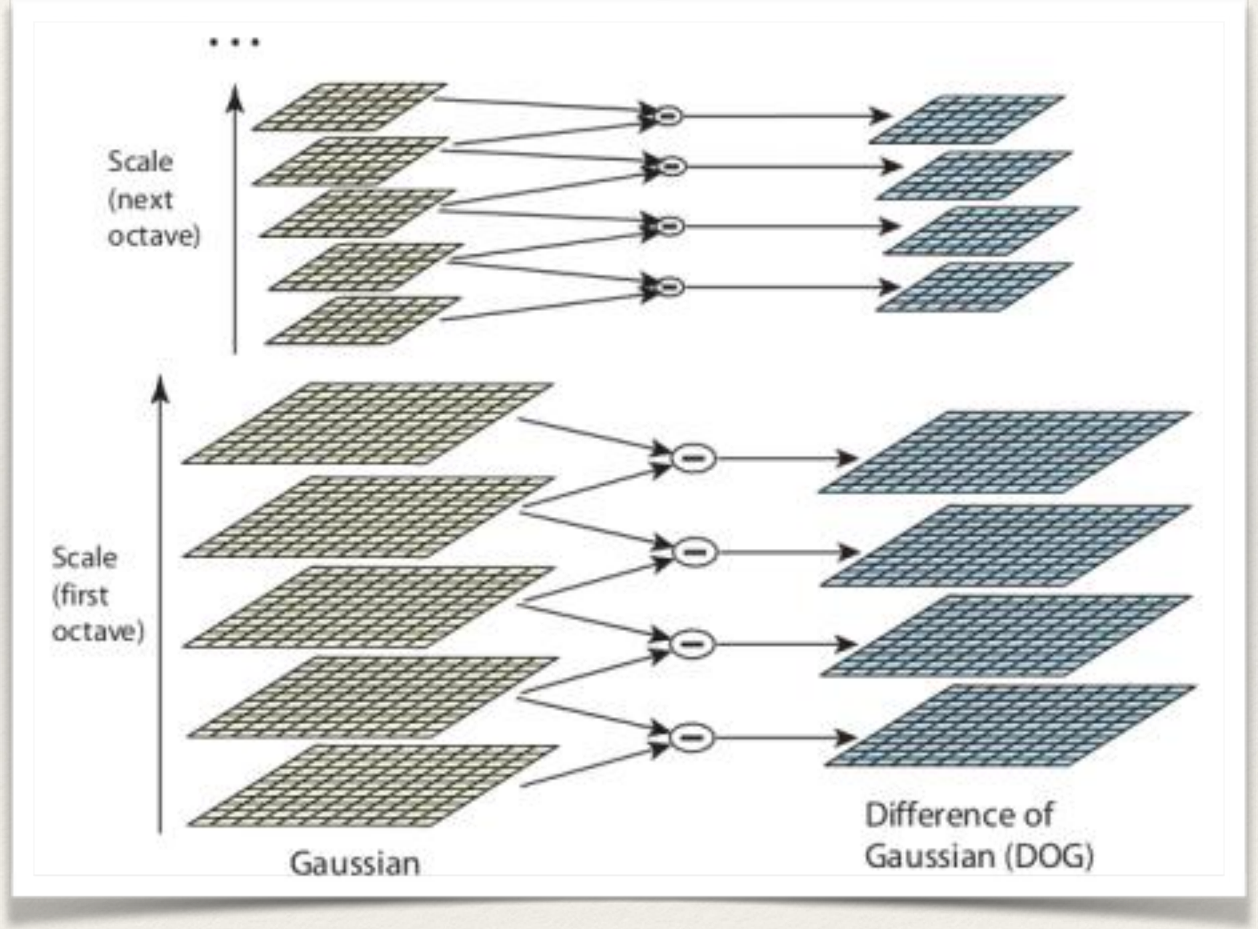
$$\nabla^2_{norm} L(x, y; t) \approx \frac{t}{\Delta t}\left(L(x, y; t + \Delta t) - L(x, y; t - \Delta t)\right)$$

- This is called a Difference-of-Gaussians (*DoG*)
  - Implies that the LoG scale space can be built from subtracting adjacent scales of a Gaussian scale space

Demo: Difference of Gaussian Response

DoG Pyramid

- Of course, for efficiency you can also build a DoG pyramid
  - an *oversampled* pyramid as there are multiple images between a doubling of scale.
  - Images between a doubling of scale are an *octave*.



Demo: Multi-scale Difference of Gaussian Blob detector

Summary

- Interest points have loads of applications in computer vision.
    - They need to be robustly detected, and invariant to rotation, lighting change, etc.
- We've looked at two types: corners and blobs
    - Harris & Stephens is a common corner detector
    - Finding extrema in a multi scale DoG pyramid provides a robust blob detector
- Scale space theory allows us to find features (corners and blobs) of different sizes