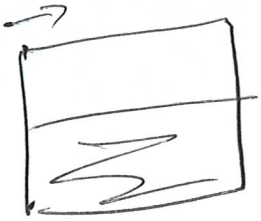
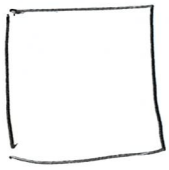
Edge detection

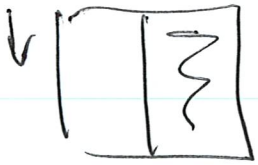
i/ differentiation ≡ differencing

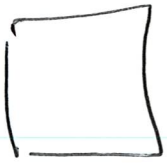
$edge_{x,y} = f_{x,y} - f_{x+1,y}$





$edge_{x,y} = f_{x,y} - f_{x,y+1}$





combine to give template

2	-1
-1	

has problems.

ii/ Better way?

Taylor series

$$A \quad f(x+\Delta x) = f(x) + \Delta x f'(x) + \frac{\Delta x^2}{2!} f''(x) + \dots$$

$$f'(x) = \frac{f(x+\Delta x) - f(x)}{\Delta x} + O(\Delta x)$$

-1	+1	✗

$$B \quad f(x-\Delta x) = f(x) - \Delta x f'(x) + \frac{\Delta x^2 f''(x)}{2!} + O(\Delta x^3)$$

$$A - B$$

$$f(x+\Delta x) - f(x-\Delta x) = 2\Delta x f'(x) + O(\Delta x^3)$$

$$f'(x) = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} + O(\Delta x^2)$$

-1	0	+1	✓

$\Delta x < 1 \quad O(\Delta x^2) \ll O(\Delta x)$

iii/ need smoothing.

diff →

av ↓

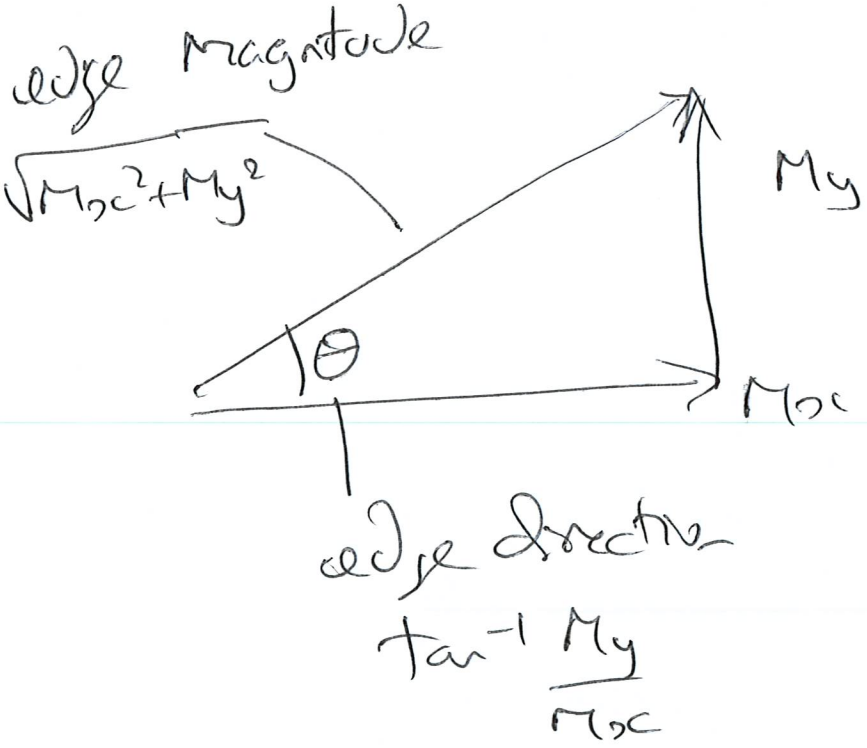
-1	0	+1
-1	0	+1
-1	0	+1

'Mx

-1	-1	-1
0	0	0
+1	+1	+1

My.

Prewitt



IV. smoothing?

```
        1   1
      1   2   1          approximation
    1   3   3   1          to
  1   4   6   4   1        Gaussian
```

∞ ↓ normal distribution

optimal smoothing.

differacy?

```
        1   -1
      1   0   -1          optimal
    1   1   -1   -1        differacy
  1   2   0   -2   -1
```

Sobel = smoothing × differacy

$$\text{smoothing}^T \times \text{differacy} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

most popular edge detection

by virtue of good performance.