Lecture 2 Image Formation

COMP3204 Computer Vision

What is inside an image?

Book
pp
29-38
57-66

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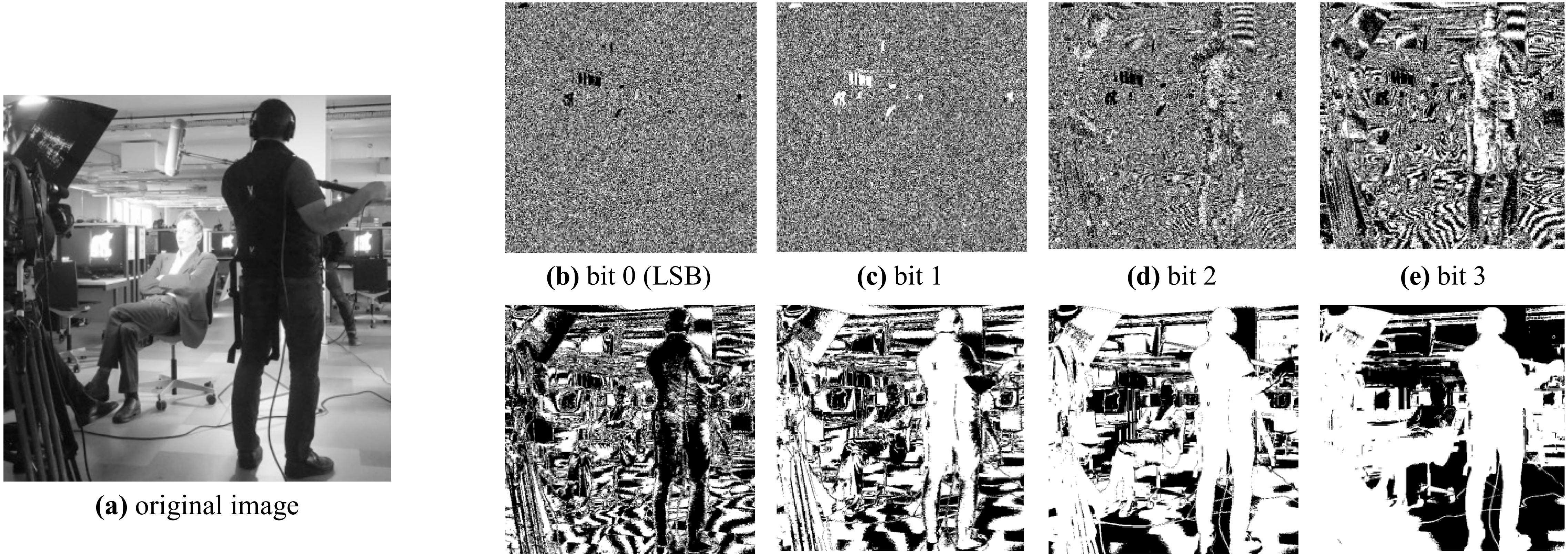
Content

1. How is an image formed?
2. What restrictions are there on image formation?
3. Go to a different space - Fourier …..

Decomposing an image into its bits

The Most Significant Bit carries the most information where as bit 0 is noise



(a) original image (b) bit 0 (LSB) (c) bit 1 (d) bit 2 (e) bit 3 (f) bit 4 (g) bit 5 (h) bit 6 (i) bit 7 (MSB)

… and here, bit 4 is the lighting

Effects of differing image resolution

(a) 64×64 **(b)** 128×128 **(c)** 256×256

Low resolution lose information but N×N points implies much storage

How do we choose an appropriate value for N?

Joseph Fourier

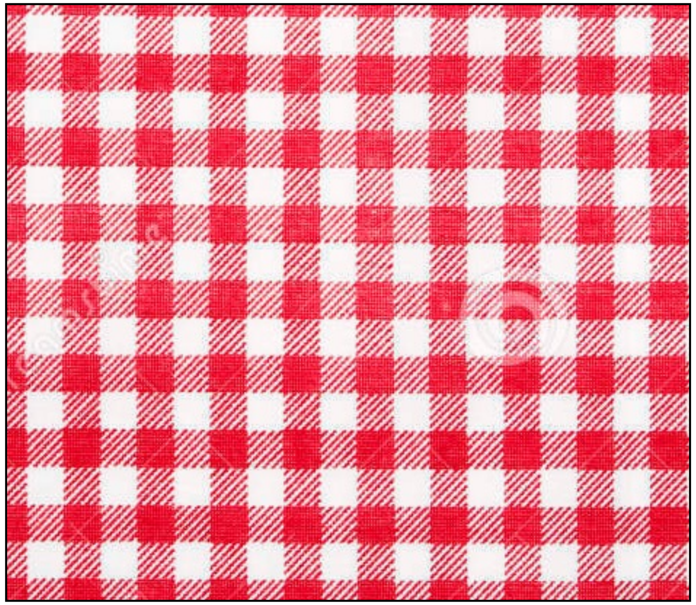
- Any periodic function is the result of adding up sine and cosine waves of different frequencies
- Sceptical? Yeah, so were Lagrange and Laplace. Good company eh?
- “Fourier’s treatise is one of the very few scientific books that can never be rendered antiquated by the progress of science” James Clerk Maxwell 1878

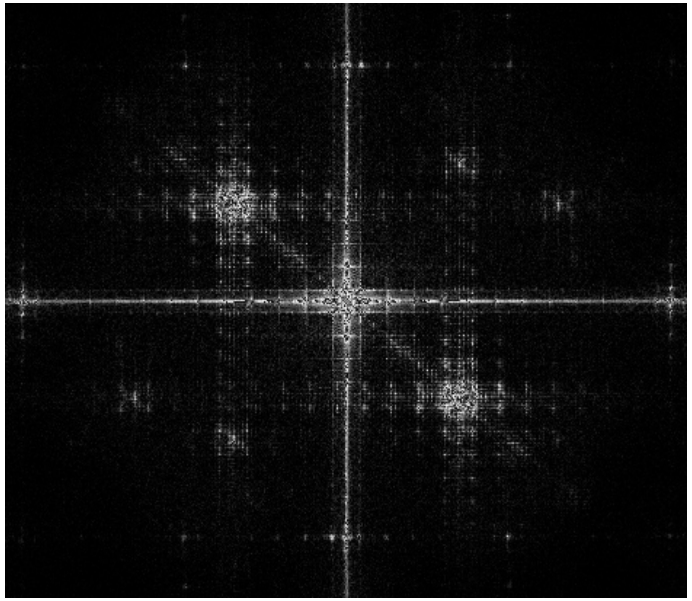
What are 2D waves?

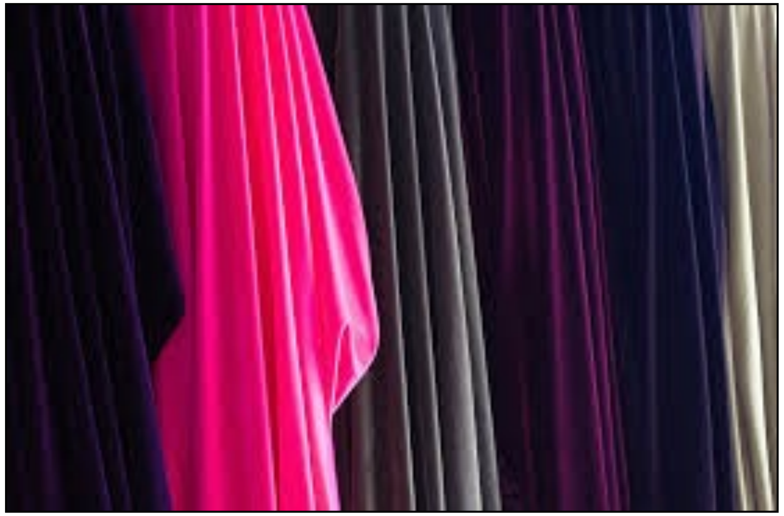
2D waves are along x and y axes simultaneously

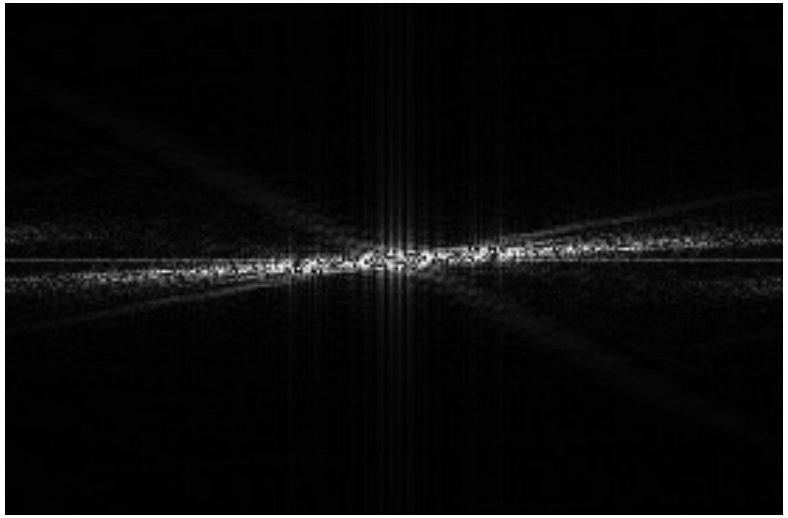
and in terms of frequency

- N.b. colour immaterial (just for visuals)









Step up Fourier…

$$Fp(f) = \mathcal{F}\big(p(t)\big) = \int_{-\infty}^{\infty} p(t)e^{-jft}dt$$

Whoa! Where from….

First, we have that the FT is a function $a(\quad)$ of a time-variant signal $p(t)$

The Fourier transform is then $\quad Fp = a\big(p(t)\big)$

The transform is a function of frequency so $\quad Fp(f) = a\big(p(t)\big)$

$\mathcal{F}$ stands for the Fourier transform so $\quad Fp(f) = \mathcal{F}\big(p(t)\big) = a\big(p(t)\big)$

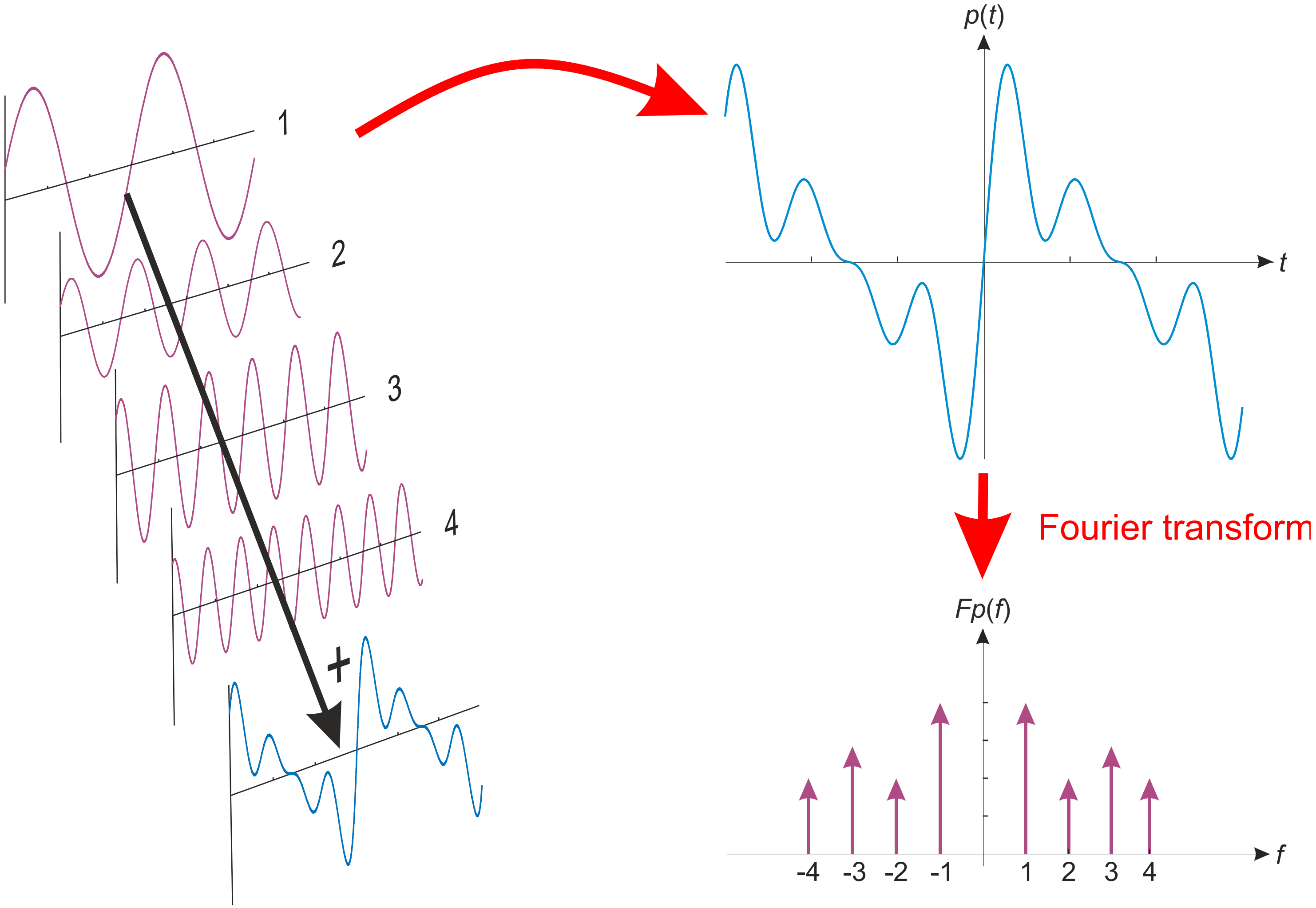
The function $a(\quad)$ is actually an integral $\quad Fp(f) = \mathcal{F}\big(p(t)\big) = \int_{-\infty}^{\infty} p(t)\mathrm{cas}(t)dt$

$\mathrm{cas}(t)$ describes cosine and sine waves, so $\quad Fp(f) = \mathcal{F}\big(p(t)\big) = \int_{-\infty}^{\infty} p(t)e^{-jft}dt$

where cas = ‘cos and sin’ since $e^{-jft} = \cos(ft) - j\sin(ft)$,

where j is the complex number $j = \sqrt{-1}$

What does the Fourier transform do?



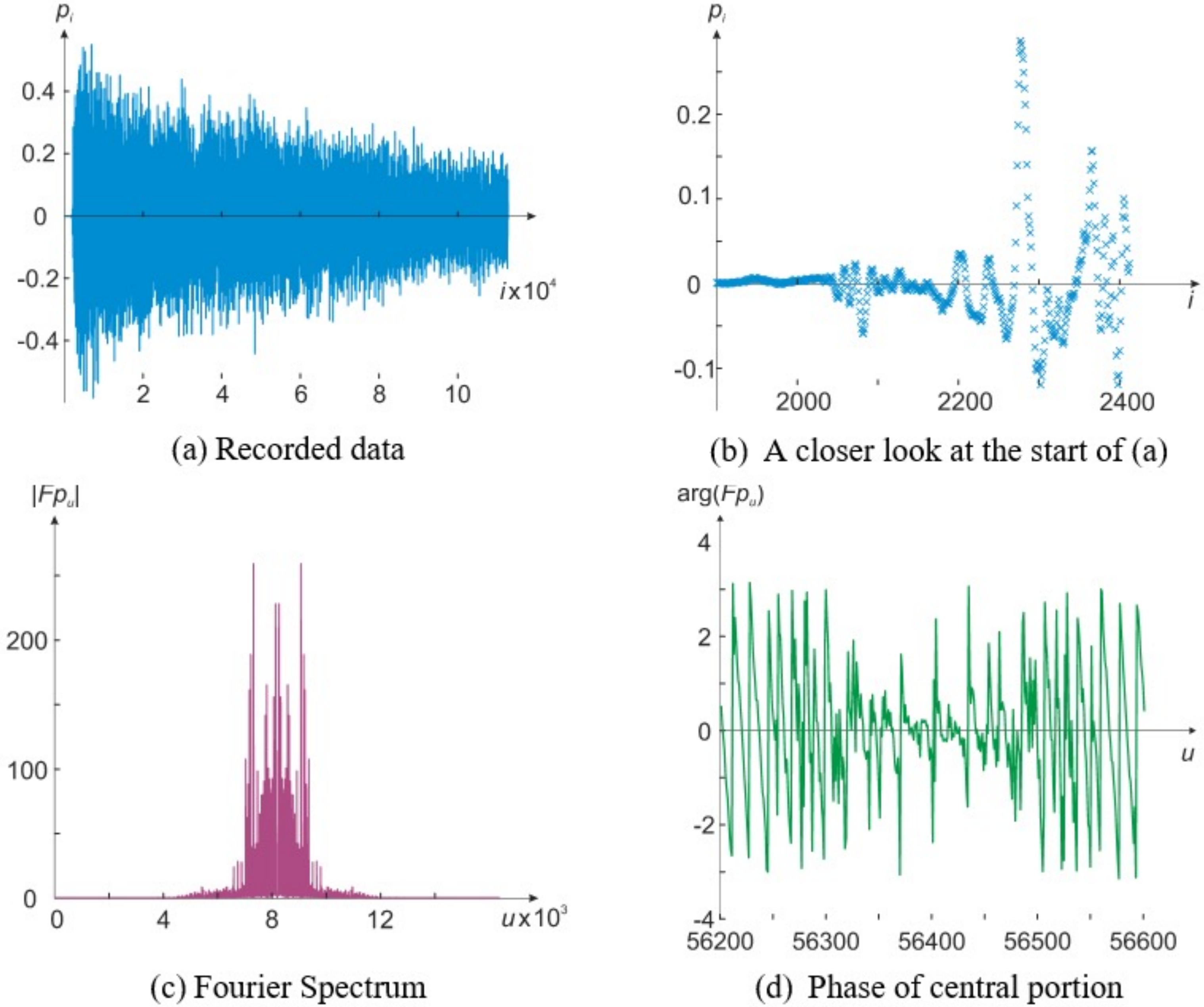
Addition in the time domain

Separation in the frequency domain

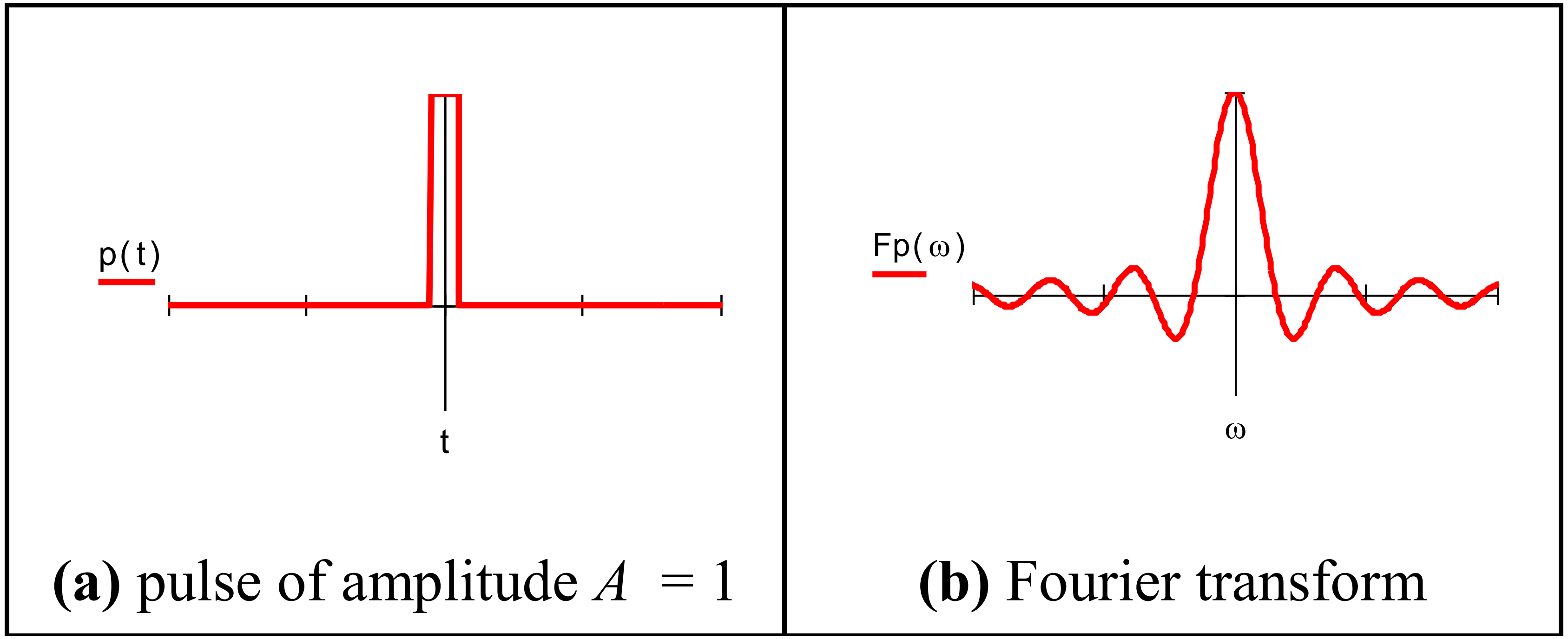
Hard day?

Let's see the Fourier transform of the Hard Day's night chord

Apparently, the piano is more dominant than expected



A rectangular pulse and its Fourier transform



(a) pulse of amplitude $A = 1$

(b) Fourier transform

- Pulse $$p(t)=\begin{cases} A & \text{if} \quad -T/2 \le t \le T/2 \\ 0 & \text{otherwise} \end{cases}$$

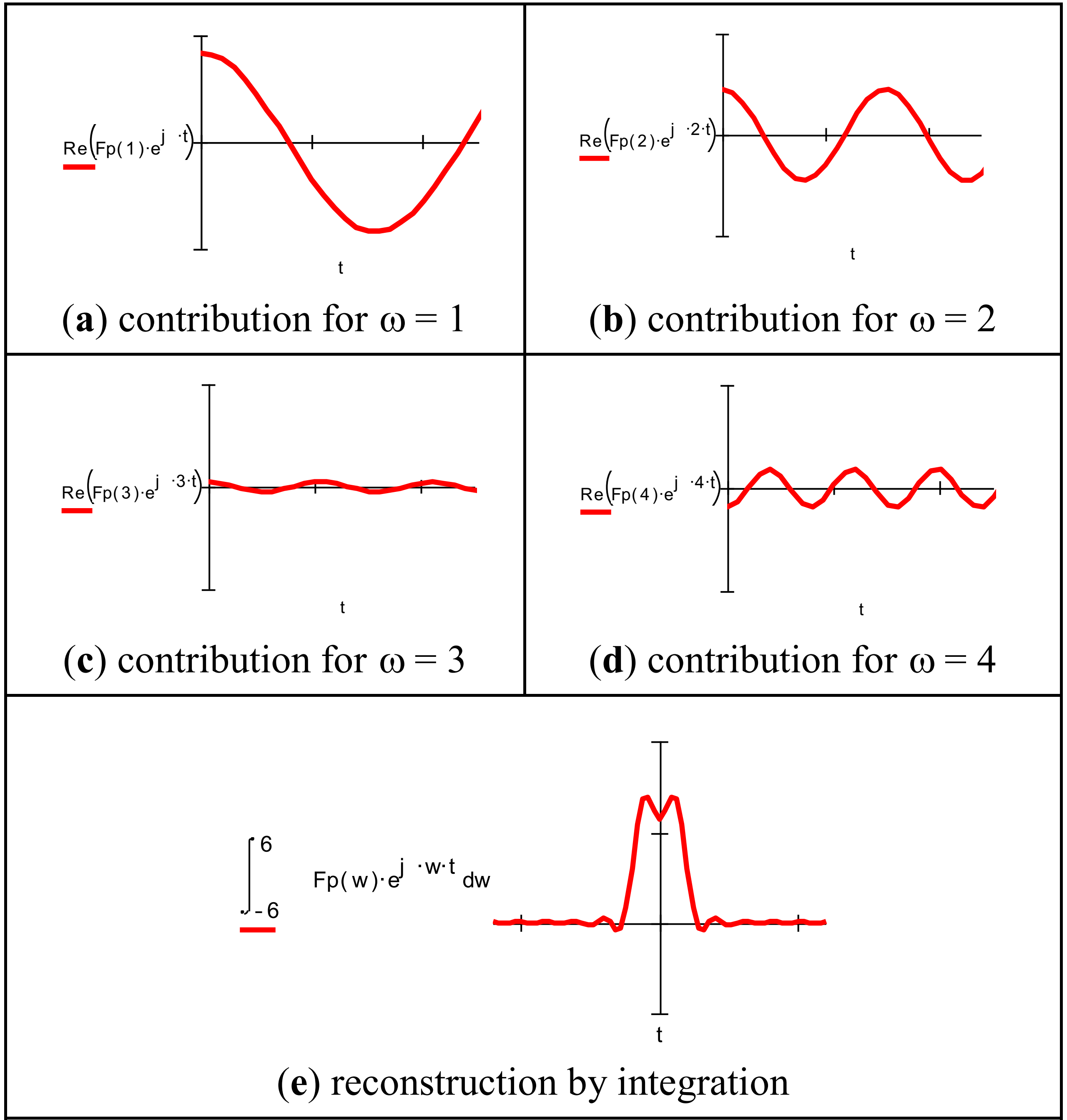
- Use Fourier $$Fp(\omega)=\int_{-T/2}^{T/2} Ae^{-j\omega t}\,dt$$

- Evaluate integral $$Fp(\omega)=-\frac{Ae^{-j\omega T/2}-Ae^{j\omega T/2}}{j\omega}$$

- And get result $$Fp(\omega)=\begin{cases} \frac{2A}{\omega}\sin\left(\frac{\omega T}{2}\right) & \text{if} \quad \omega \ne 0 \\ AT \quad \text{if} \quad \omega = 0 \end{cases}$$

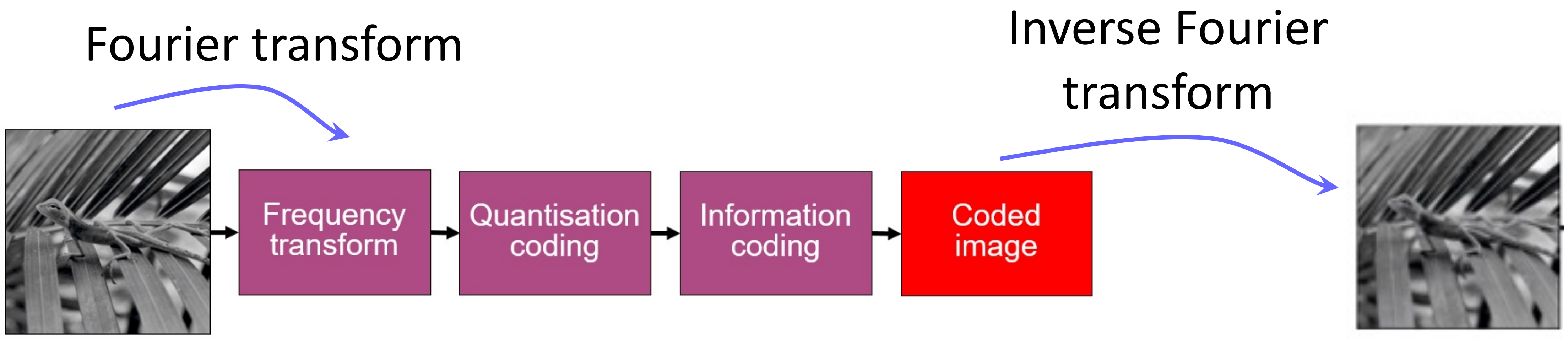
Reconstructing a signal from its Fourier transform

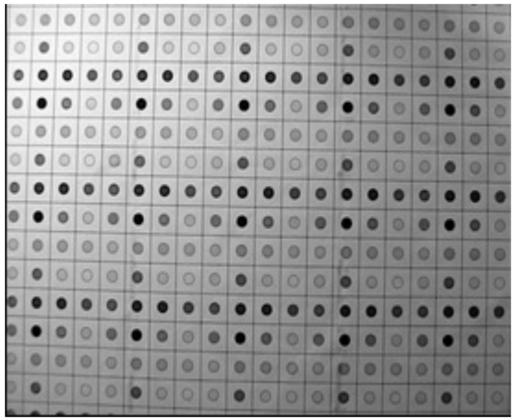
This is the **inverse Fourier transform**



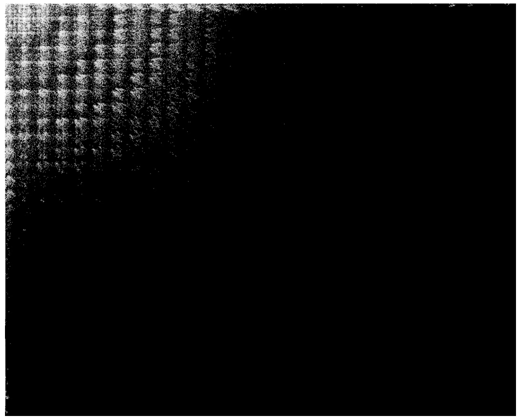
Reconstructing a Signal from its Transform

Inverse Fourier transform is used for reconstruction

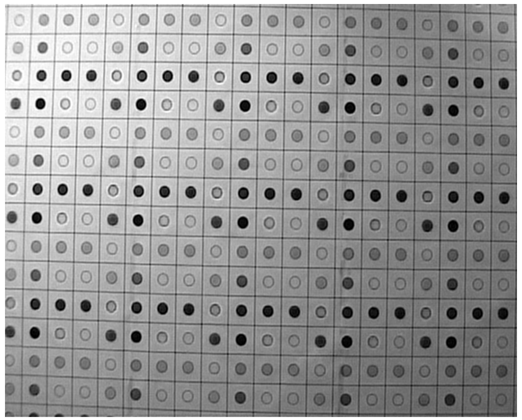




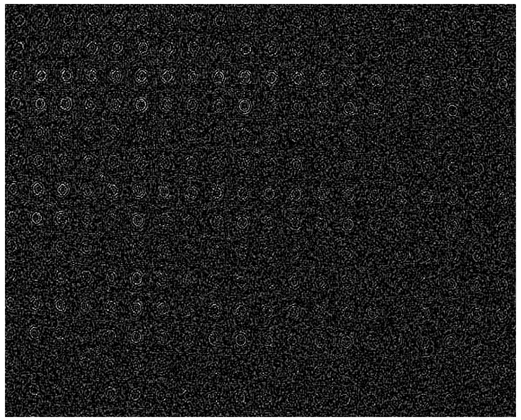
image



5% of transform components



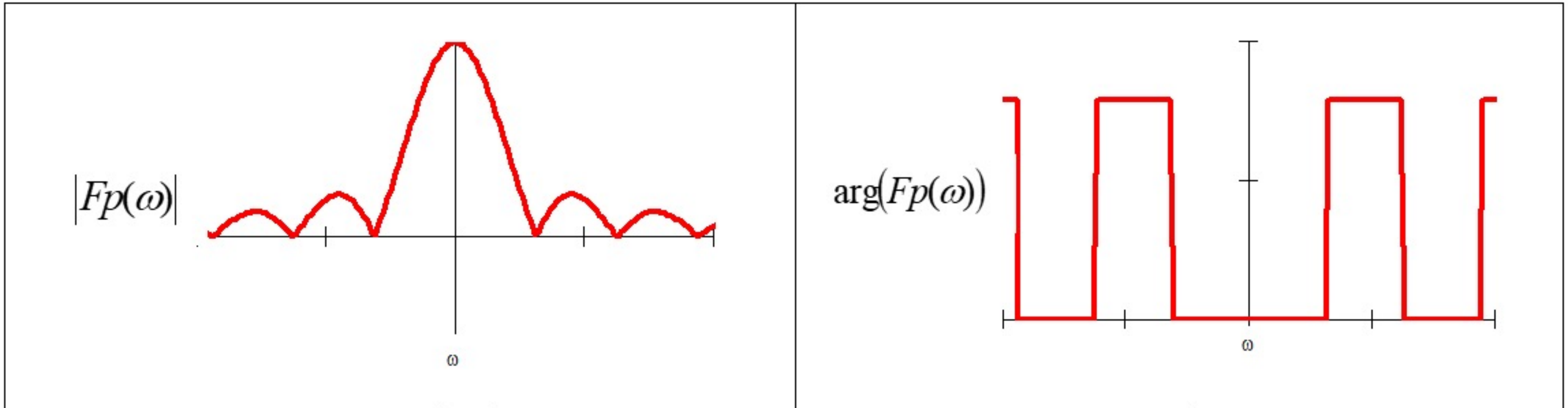
reconstructed image



error

Magnitude and phase of Fourier transform of a pulse

$$Fp(\omega) = \int_{-\infty}^{\infty} p(t)e^{-j\omega t}dt = \mathrm{Re}\left(Fp(\omega)\right) + j\,\mathrm{Im}\left(Fp(\omega)\right)$$

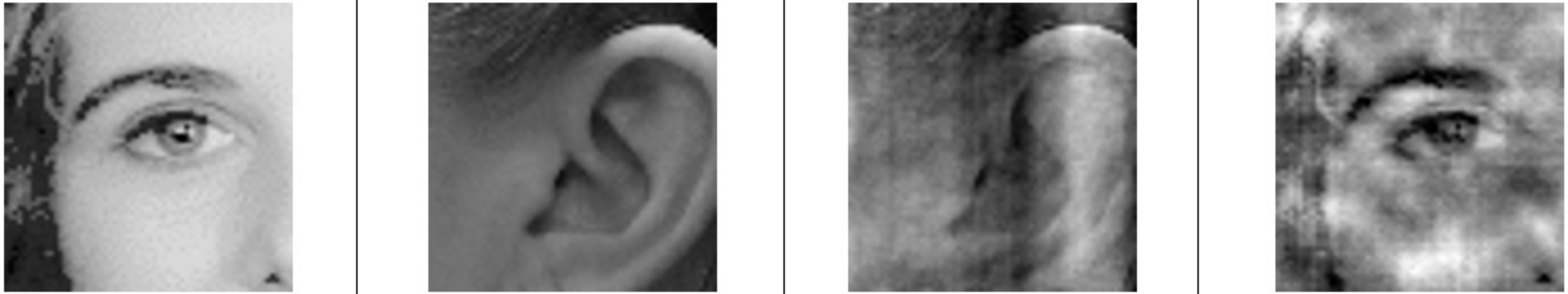


(a) magnitude **(b)** phase

$$|Fp(\omega)| = \sqrt{\mathrm{Re}\left(Fp(\omega)\right)^2 + \mathrm{Im}\left(Fp(\omega)\right)^2}$$

$$\arg\left(Fp(\omega)\right) = \tan^{-1}\left(\frac{\mathrm{Im}\left(Fp(\omega)\right)}{\mathrm{Re}\left(Fp(\omega)\right)}\right)$$

Illustrating the importance of phase



(a) eye image	**(b)** ear image	**(c)** reconstruction from magnitude(eye) and phase(ear)	**(d)** reconstruction from magnitude(ear) and phase(eye)

Main points so far

1 – sampling data is not as simple as it appears

2 – sampling affects space and brightness

3 – Fourier allows us to understand frequency

4 – Fourier allows for coding and more

Next, Fourier will allow us to understand
sampling

Other transforms

- Discrete Cosine (Sine) Transform
- Discrete Hartley Transform

Wavelets

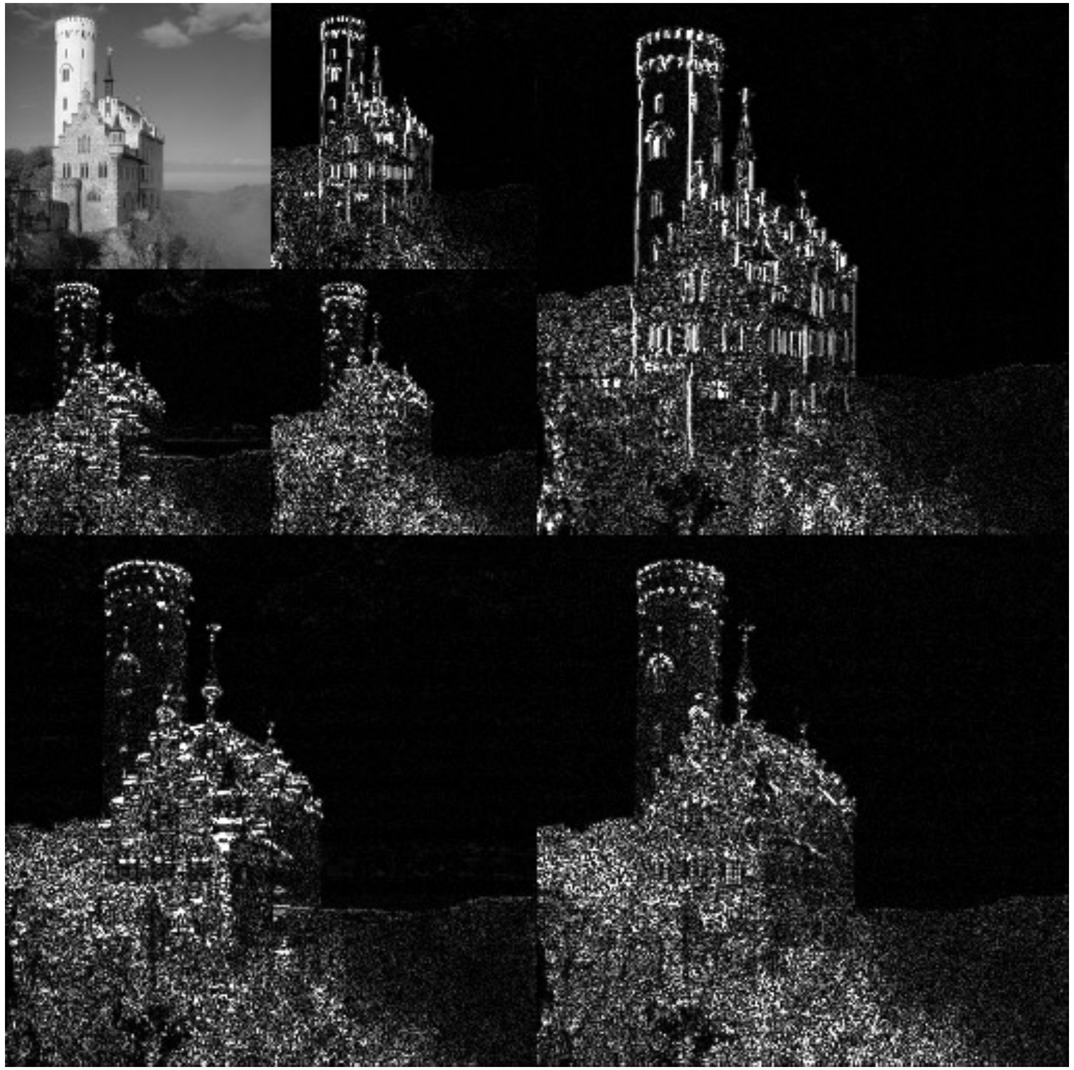
Continuous wavelet
Discrete wavelet
Complex wavelet
Stationary wavelet
Dual wavelet
Haar wavelet
Daubechies wavelet
Morlet wavelet
Gabor wavelet
……

Curvelets

Shearlets

Bandelet
Contourlet
Fresnelet
Chirplet
Noiselet
……

Wavelet transform



An example of the 2D discrete wavelet transform that is used in JPEG2000 [Credit: Wikipedia]