Lecture 3 Image Sampling

COMP3204 Computer Vision

How is an image sampled and what does it imply?

Book
pp
36-56

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Content

1. How does the discrete Fourier transform work, and help?
2. What can go wrong with sampling?

1D Discrete Fourier transfrom

Discrete Fourier calculates frequency from data points

$$Fp_u = \frac{1}{N}\sum_{i=0}^{N-1} p_i e^{-j\frac{2\pi}{N}iu}$$

Comparison

$$Fp(\omega) = \int_{-\infty}^{\infty} p(t)e^{-j\omega t}dt$$

sampled frequency Fp_u

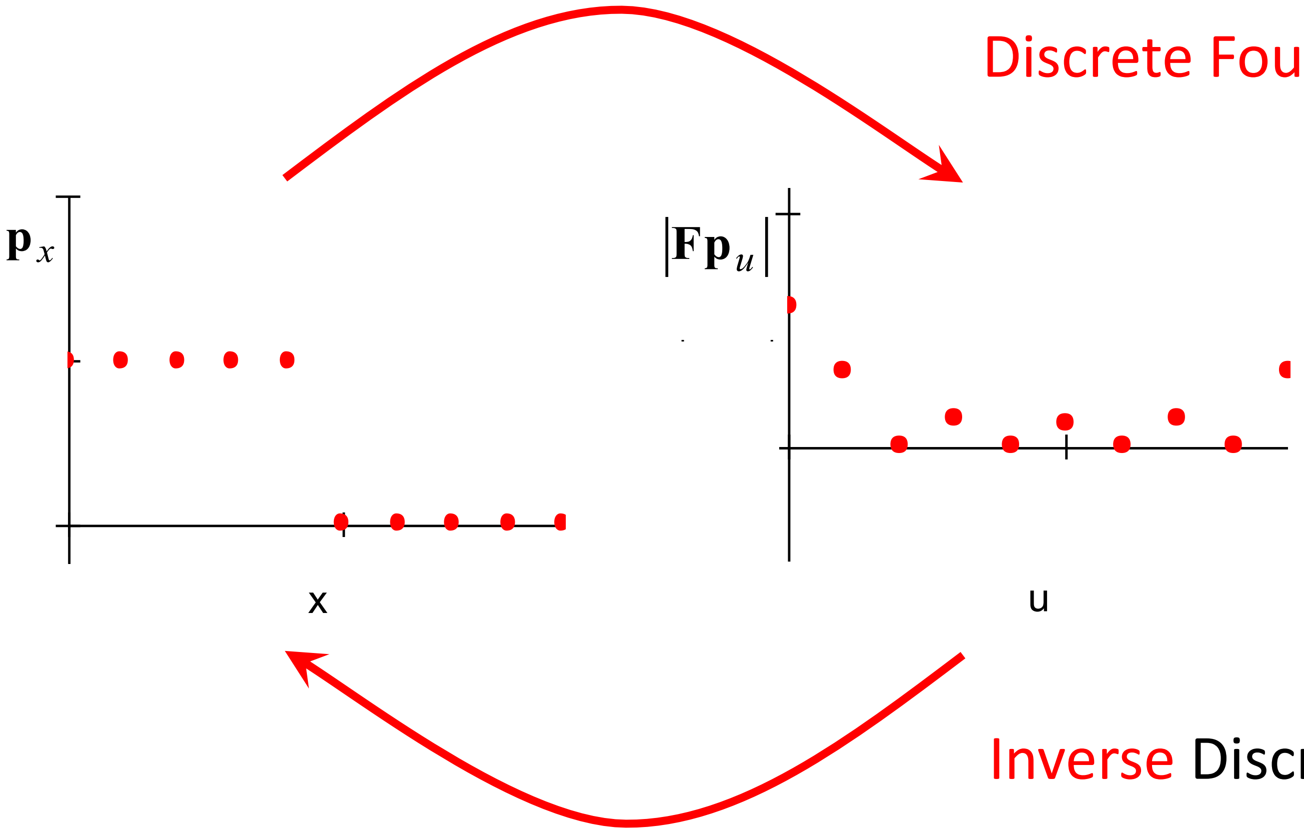
sampled points p_i

N points

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

Transform Pair for Sampled Pulse

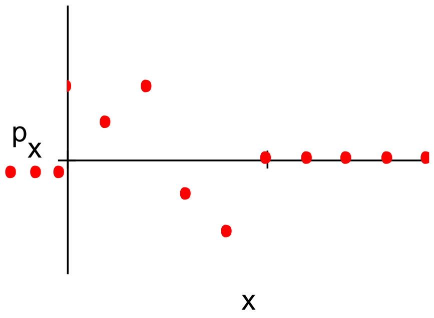
Discrete Fourier transform



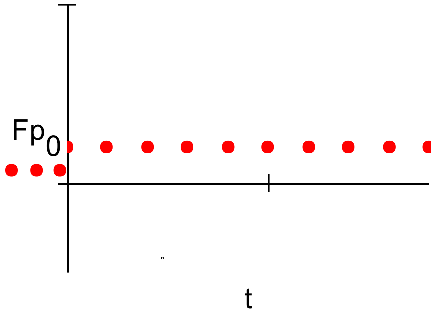
Inverse Discrete Fourier transform

(a) sampled pulse

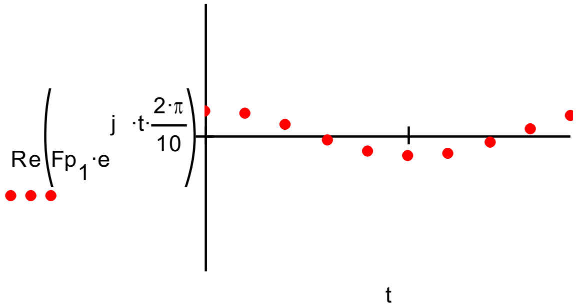
(b) DFT of sampled pulse



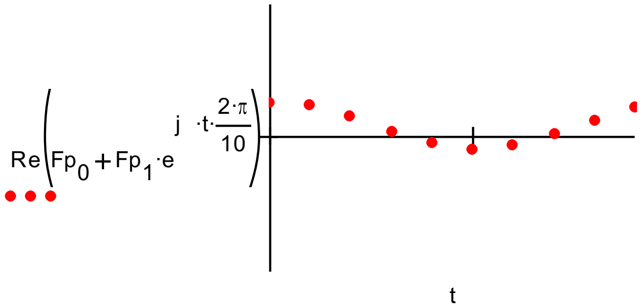
(a) original sampled signal



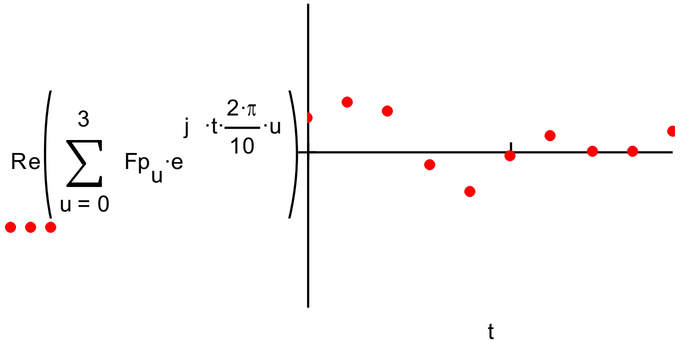
(b) first coefficient Fp_0



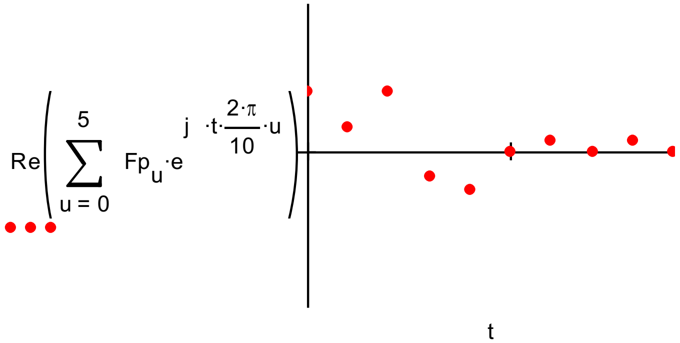
(c) second coefficient Fp_1



(d) adding Fp_1 and Fp_0



(e) adding Fp_0, Fp_1 , Fp_2 and Fp_3



(f) adding all six frequency components

signal reconstruction from its transform components

2D Fourier transform

Forward transform

$$\mathbf{FP}_{u,v} = \frac{1}{N^2} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \mathbf{P}_{x,y} e^{-j\left(\frac{2\pi}{N}\right)(ux+vy)}$$

where two dimensions of space, *x* and *y*

two dimensions of frequency, *u* and *v*

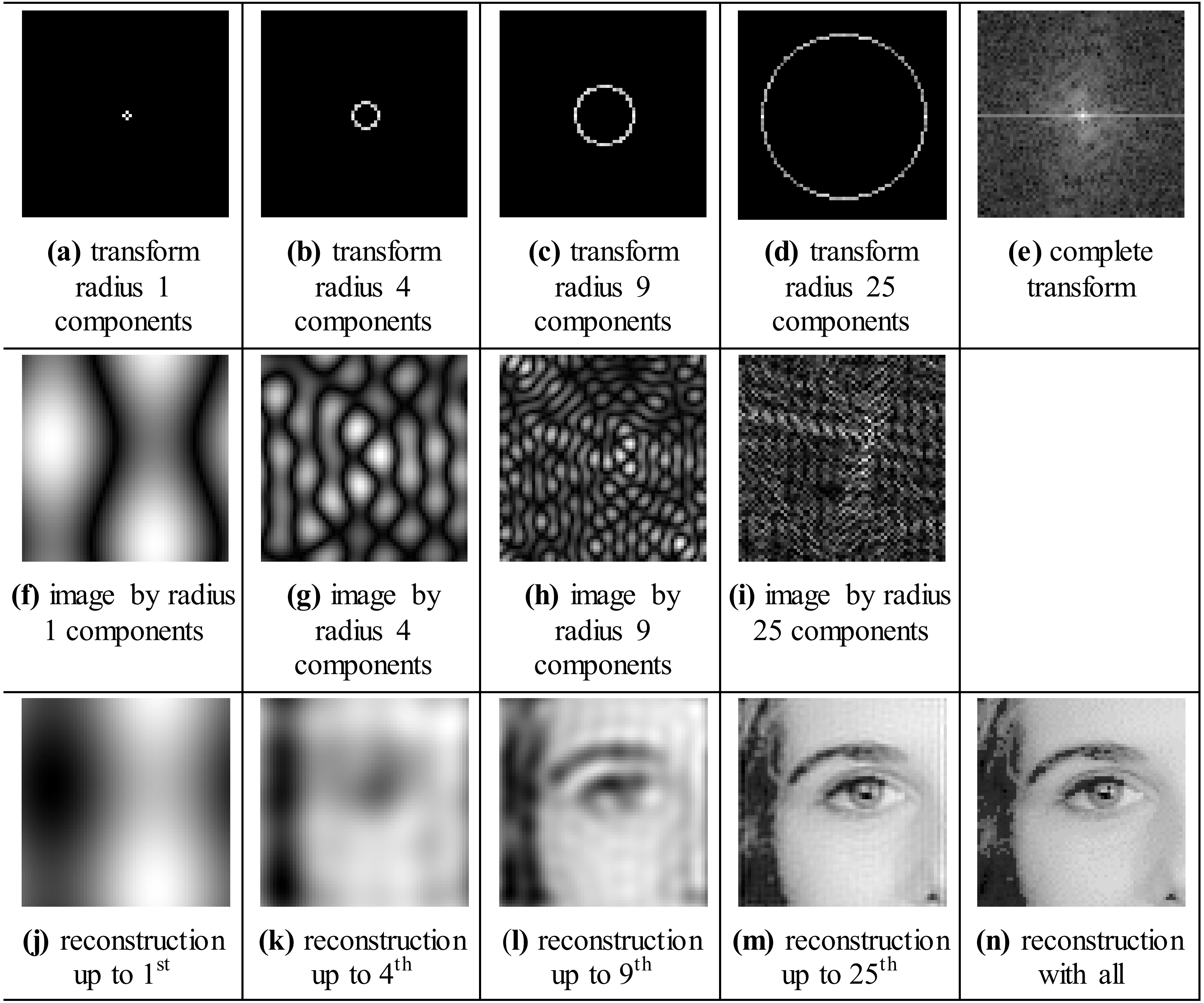
image *N*x*N* pixels $\mathbf{P}_{x,y}$

Inverse transform

$$\mathbf{P}_{x,y} = \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} \mathbf{FP}_{u,v} e^{j\left(\frac{2\pi}{N}\right)(ux+vy)}$$

π??

Reconstruction



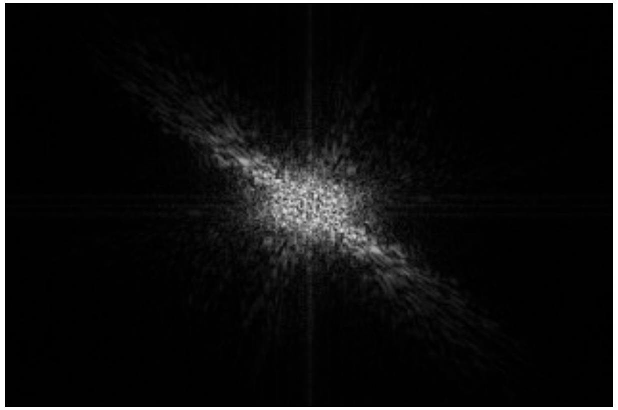
Implementation is via (Fast) FFT

```
while L<cols %iterate until log2(cols)-1 levels have been performed
  for j=1:2*L:cols %do all the points in L/2 batches
    for i=1:L %now do L butterflies
      upp(((j+1)/2)+i-1)= Fp(j+i-1)+Fp(j+L+i-1)*exp(-1j*2*pi*(i-1)/(L*2));
      low(((j+1)/2)+i-1)= Fp(j+i-1)-Fp(j+L+i-1)*exp(-1j*2*pi*(i-1)/(L*2));
    end
  end
  for j=1:2*L:cols %copy the components across, to the right places
    for i=1:L
      Fp(j+i-1)=upp(((j+1)/2)+i-1);
      Fp(j+L+i-1)=low(((j+1)/2)+i-1);
    end
  end
L=L*2; %and go and do the next level (up)
end
```

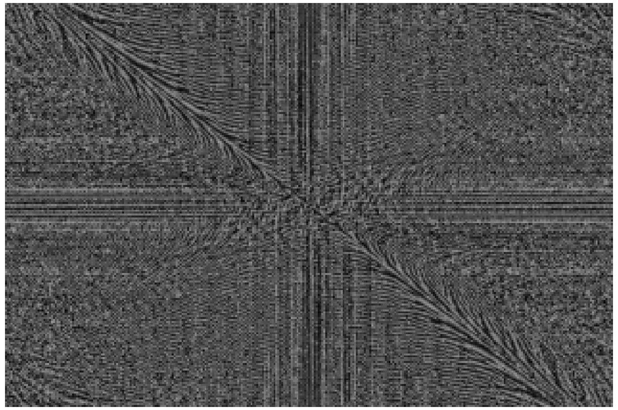
(this is a 1-D FFT)

Shift invariance

(a) original image

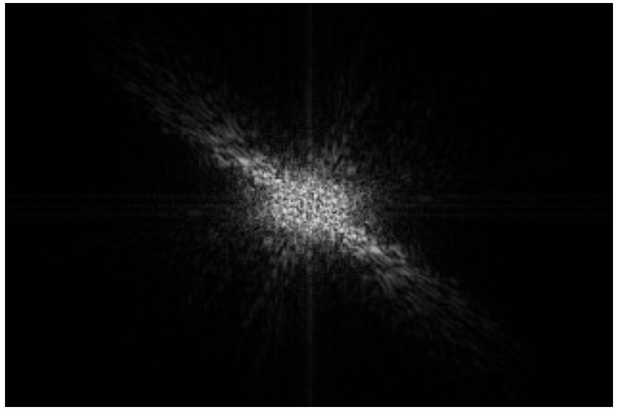


(b) magnitude of Fourier transform of original image

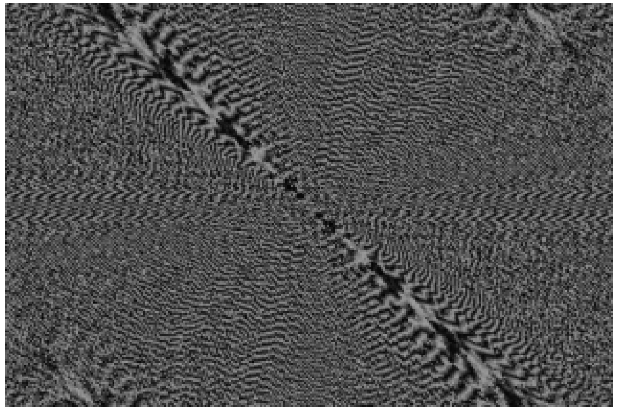


(c) phase of Fourier transform of original image

(d) shifted image

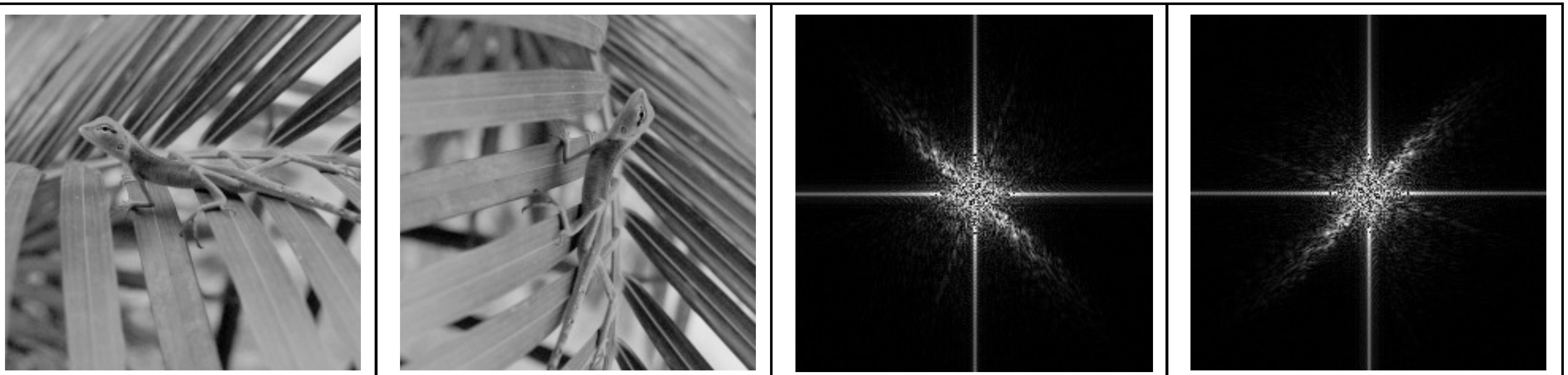


(e) magnitude of Fourier transform of shifted image



(f) phase of Fourier transform of shifted image

Rotation

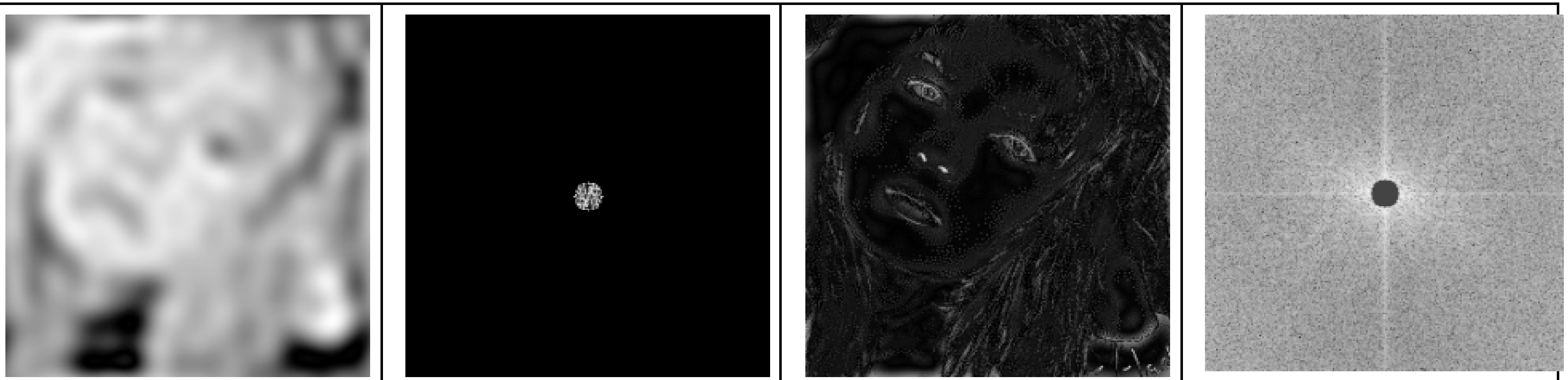


(a) original image | **(b)** rotated image | **(c)** transform of original image | **(d)** transform of rotated image

$$\mathbf{FP}_{u,v} = \frac{1}{N}\sum_{y=0}^{N-1}\sum_{x=0}^{N-1}\mathbf{P}_{x,y}\,e^{-j\left(\frac{2\pi}{N}\right)(uy+vx)}$$

Filtering

Fourier gives access to frequency components



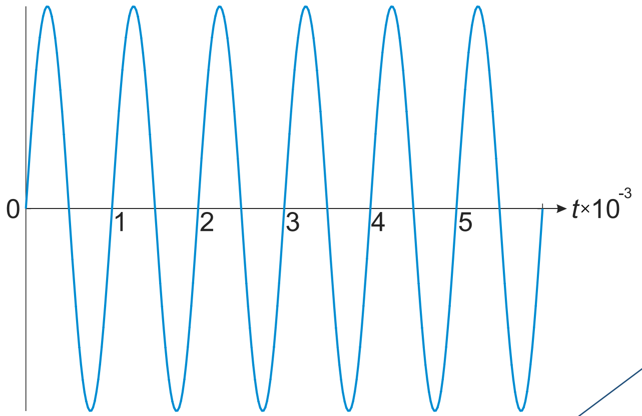
(a) low-pass filtered image | (b) low-pass filtered transform | (c) high-pass filtered image | (d) high-pass filtered transform

Applications of 2D FT

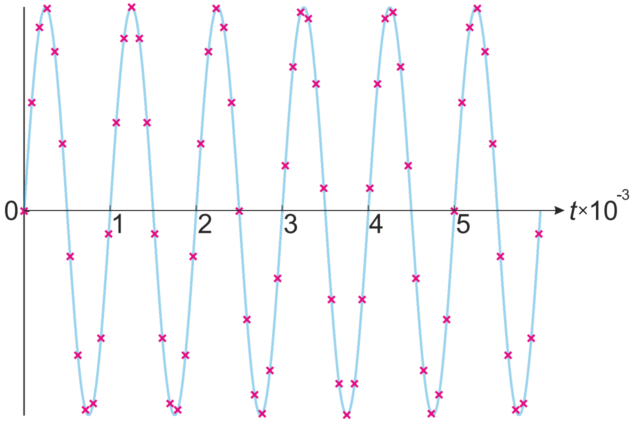
- Understanding and analysis
- Speeding up algorithms
- Representation (invariance)
- Coding
- Recognition/ understanding (e.g. texture)

Sampling Signals

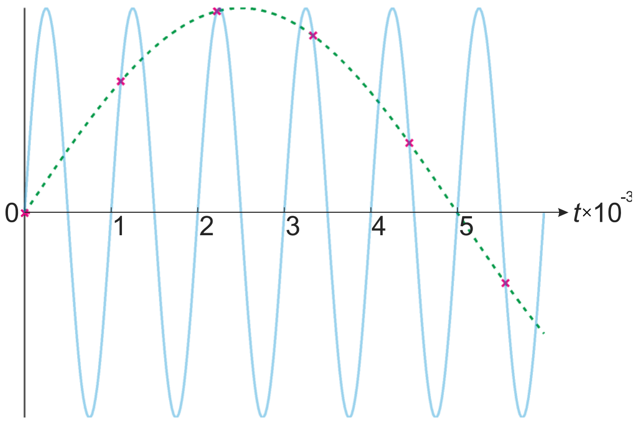
original continuous signal

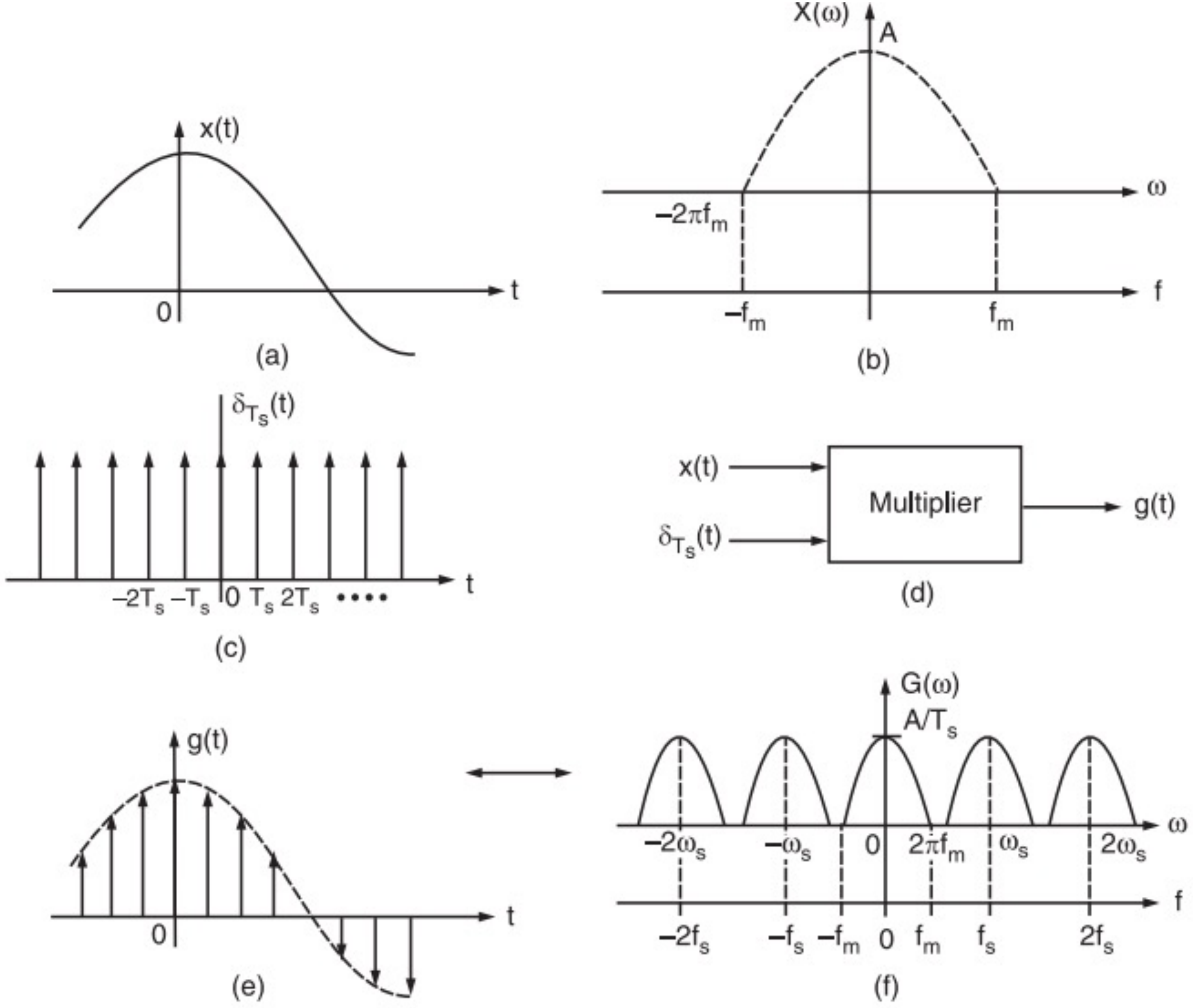


good sampling



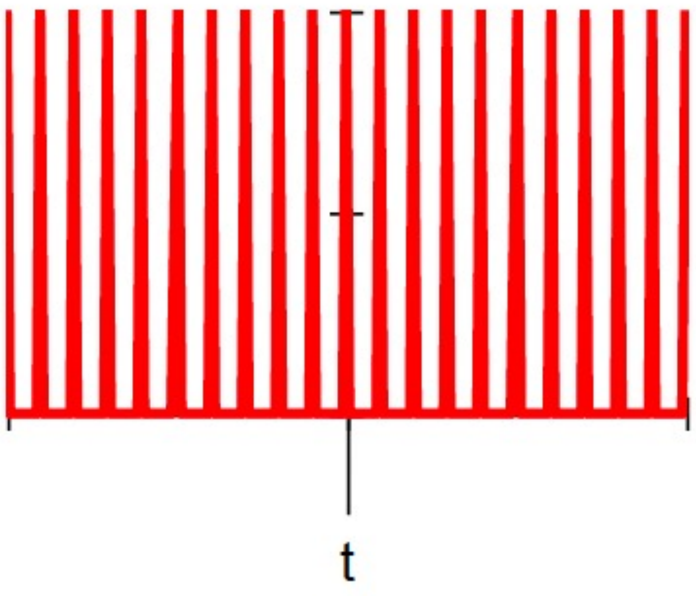
bad sampling (aliased)



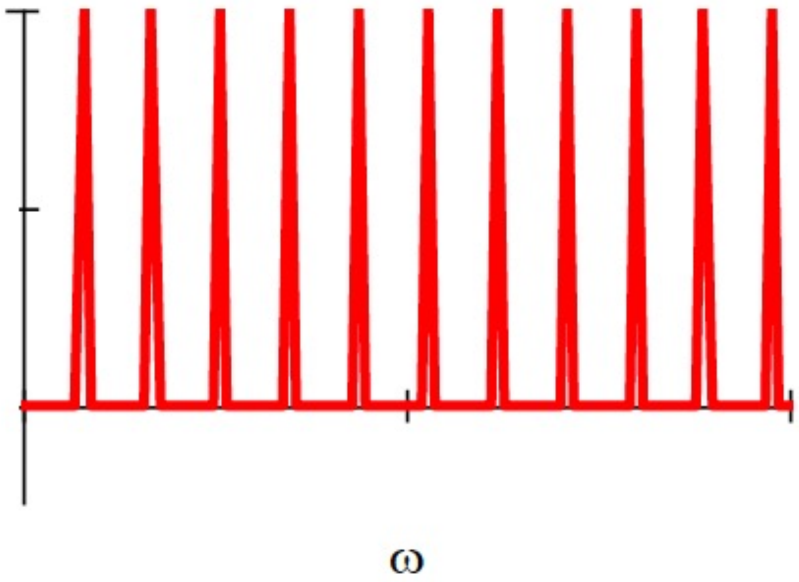


https://electronicspost.com/sampling-theorem/

Sampling function



Sampling function in
time domain



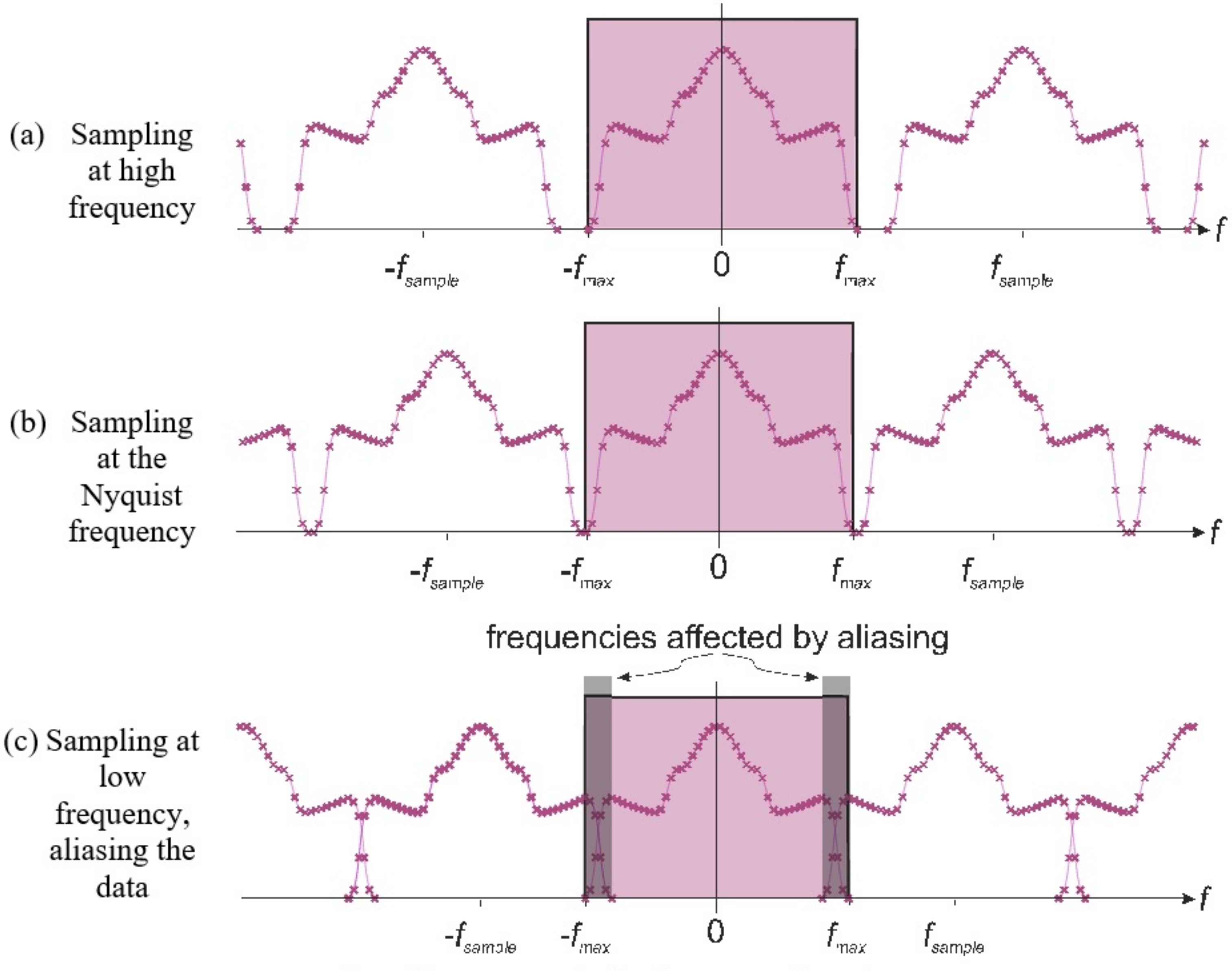
Sampling function in
frequency domain - fft

In the frequency domain

Spectra repeat

If sampling is just right, spectra just touch

Minimum sampling frequency = 2 × max



Sampling process in the frequency domain

Sampling theory

Nyquist's sampling theorem

In order to be able to reconstruct a signal from its samples we must sample at minimum at twice the maximum frequency in the original signal

E.g. speech 6kHz, sample at 12 kHz

Video bandwidth (CCIR) is 5MHz

Sampling at 10MHz gave 576×576 images

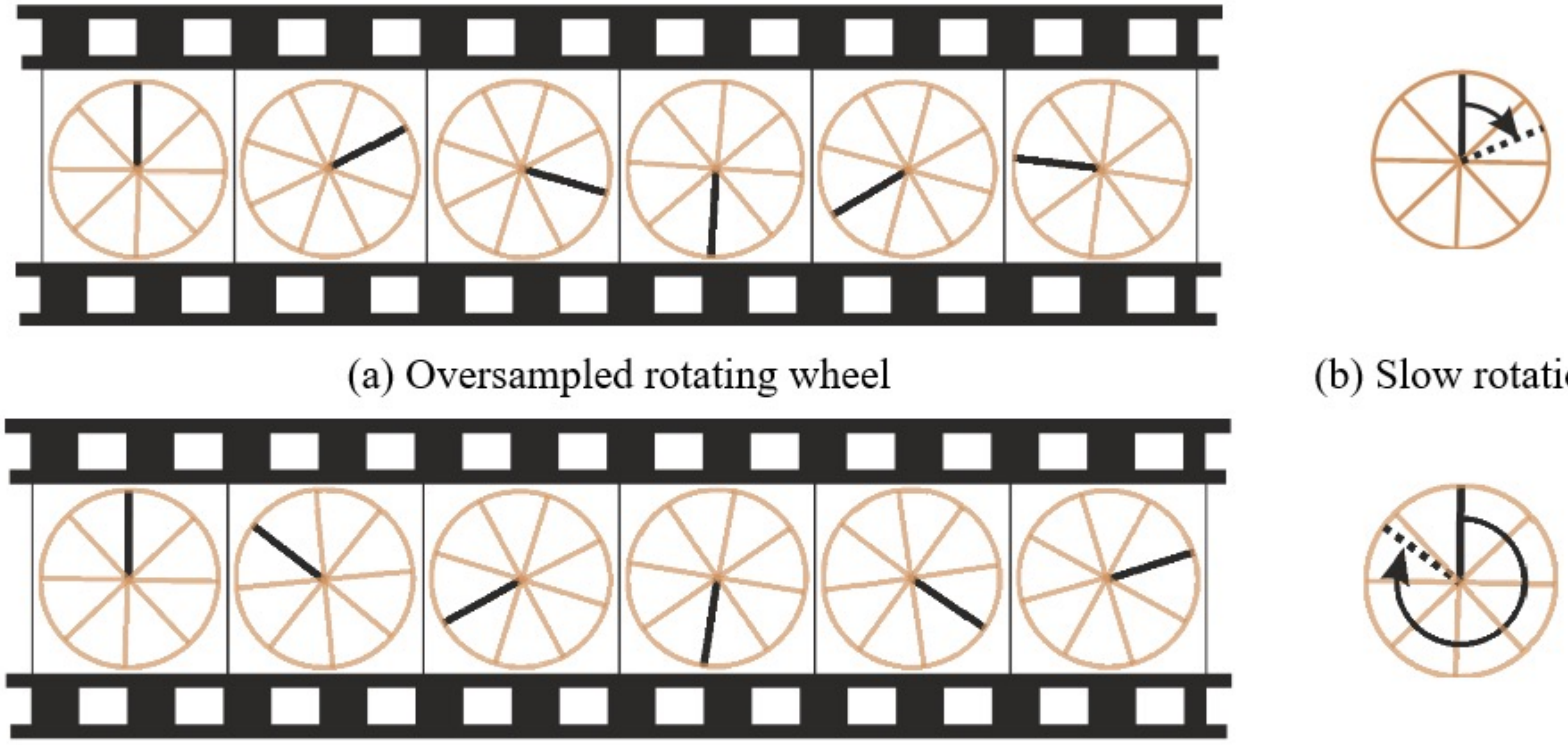
Guideline: "two pixels for every pixel of interest"

Aliasing in Sampled Imagery

(a) high resolution

(c) low resolution – aliased

Correct and Incorrect Apparent Wheel Motion



(a) Oversampled rotating wheel

(b) Slow rotation

(c) Undersampled rotating wheel

(d) Fast rotation

Figure 4.5 Correct and incorrect apparent wheel motion

https://www.youtube.com/watch?v=e1EqXE06xr8

Main points so far

1 – need to sample at a high enough frequency

2 – aliasing corrupts image information

3 – discrete Fourier allows analysis and understanding

4 – Fourier has many properties and advantages

…. but it's complex. So we'll move on to processing images

More sampling theories

Compressed sensing

Many signals are sparse…

Regularisation