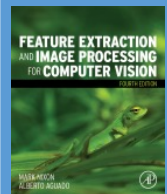


# Lecture 9 Finding More Shapes

COMP3204 Computer Vision

**How can we go from conic sections to general shapes?**



Book  
pp  
215-246

Department of  
Electronics and  
Computer Science

UNIVERSITY OF  
**Southampton**  
School of Electronics  
and Computer Science

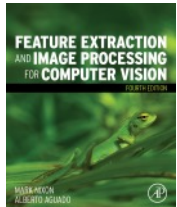
# Content

1. What more versions of the Hough transform are possible?
2. What are its limits?
3. Can it be used to detect shapes that are not given by an equation?

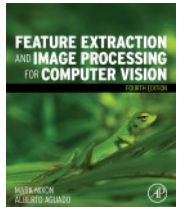
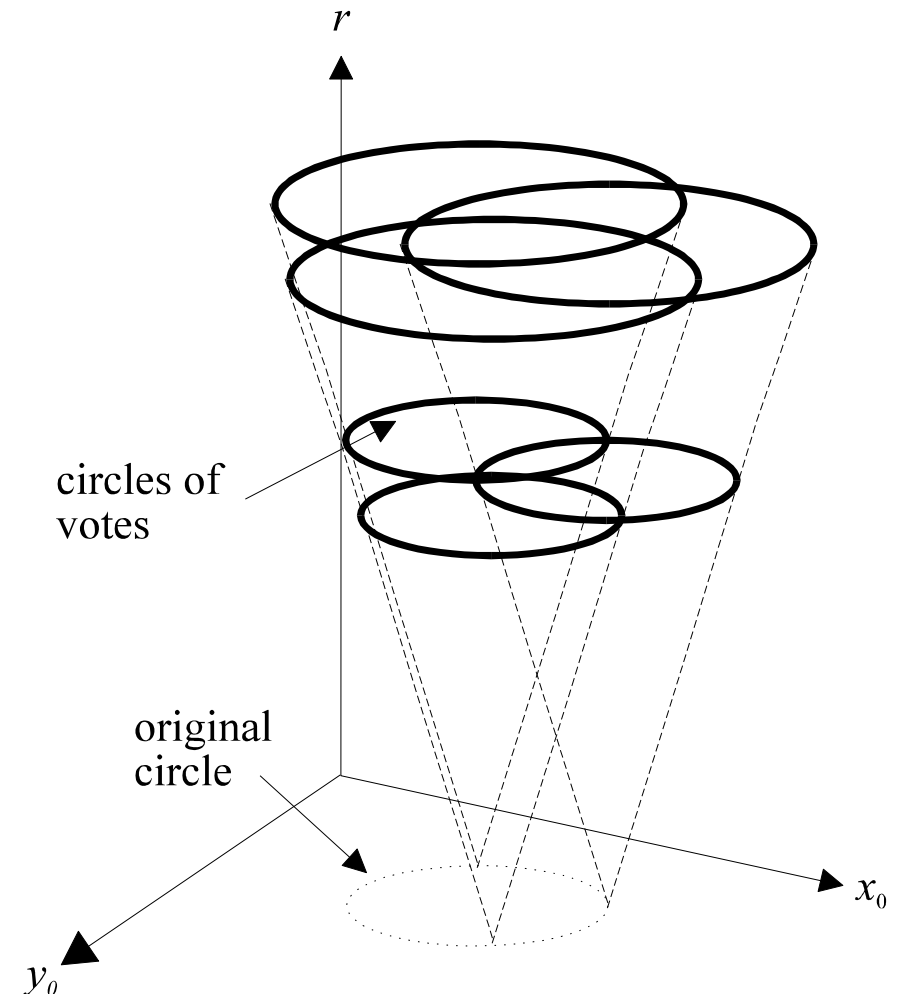
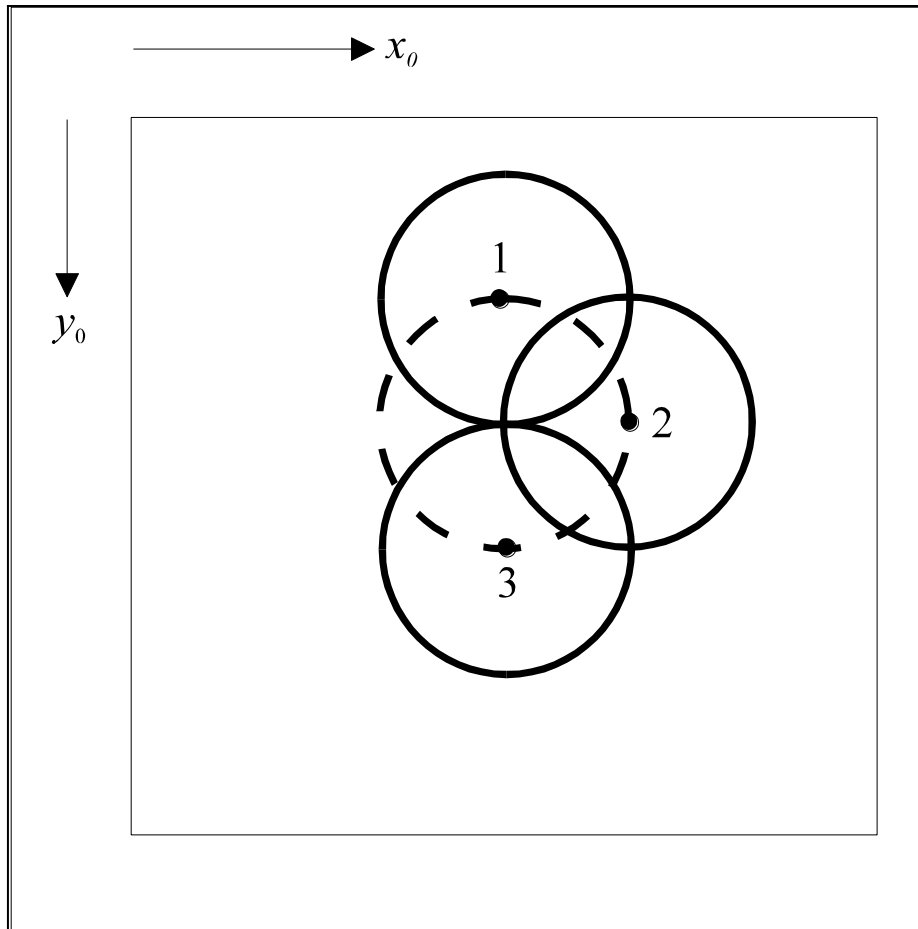
# Hough Transform for Circles

Again, it's **duality**:  $(x - x_0)^2 + (y - y_0)^2 = r^2$

Points:  $x, y$    centre:  $x_0, y_0$    radius:  $r$   
"    $x_0, y_0$    "    $x, y$    "    $r$



# Circle Voting and Accumulator Space



# Pseudocode

```
accum=0
```

```
for all x,y
```

```
    if edge(y,x)>threshold
```

```
        for r=min_r, max_r
```

```
            for theta = 0,2*pi
```

```
                x0=x+r*cos(theta)
```

```
                y0=y+r*sin(theta)
```

```
                accum(y0, x0, r ) PLUS 1      !vote in accumulator
```

```
y0, x0, r = argmax(accum)
```

```
!look at all points
```

```
!check significance
```

```
!do values of radius
```

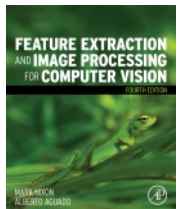
```
!go around a circle
```

```
!generate x
```

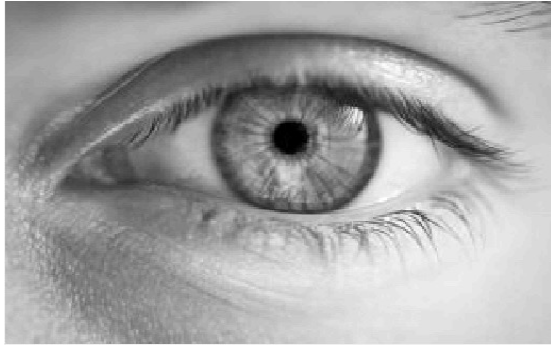
```
!generate y
```

```
!vote in accumulator
```

```
!peak gives parameters
```



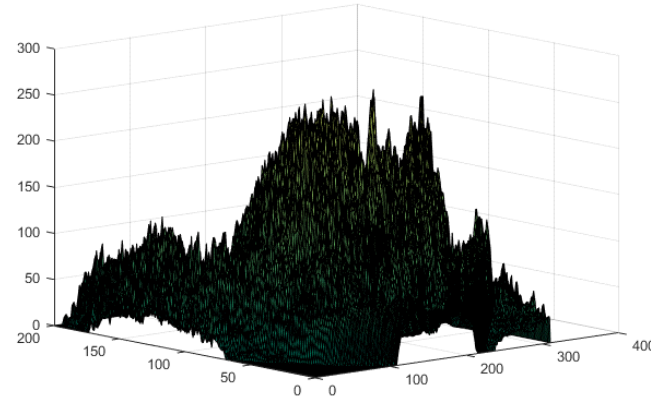
# Applying the HT for circles



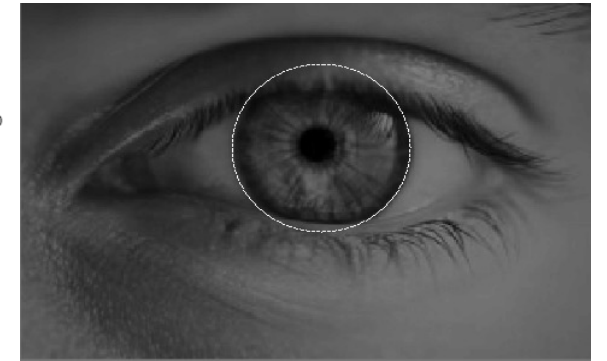
image



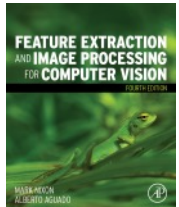
(Sobel) edges



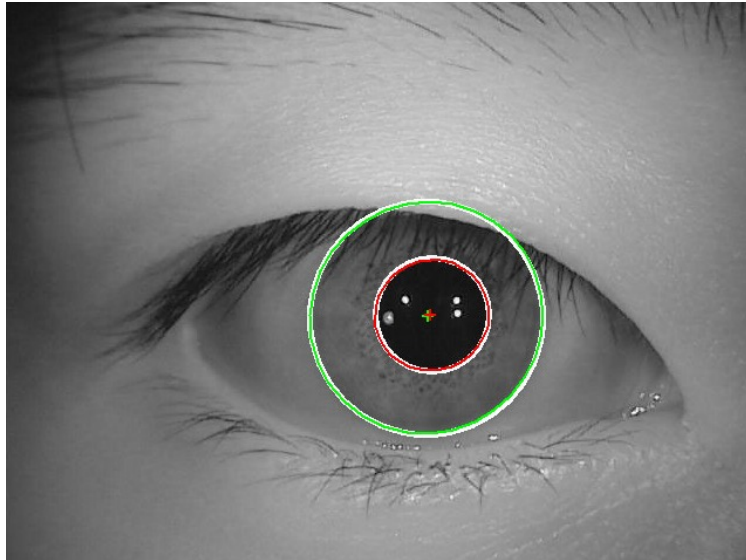
accumulator



small and large circles

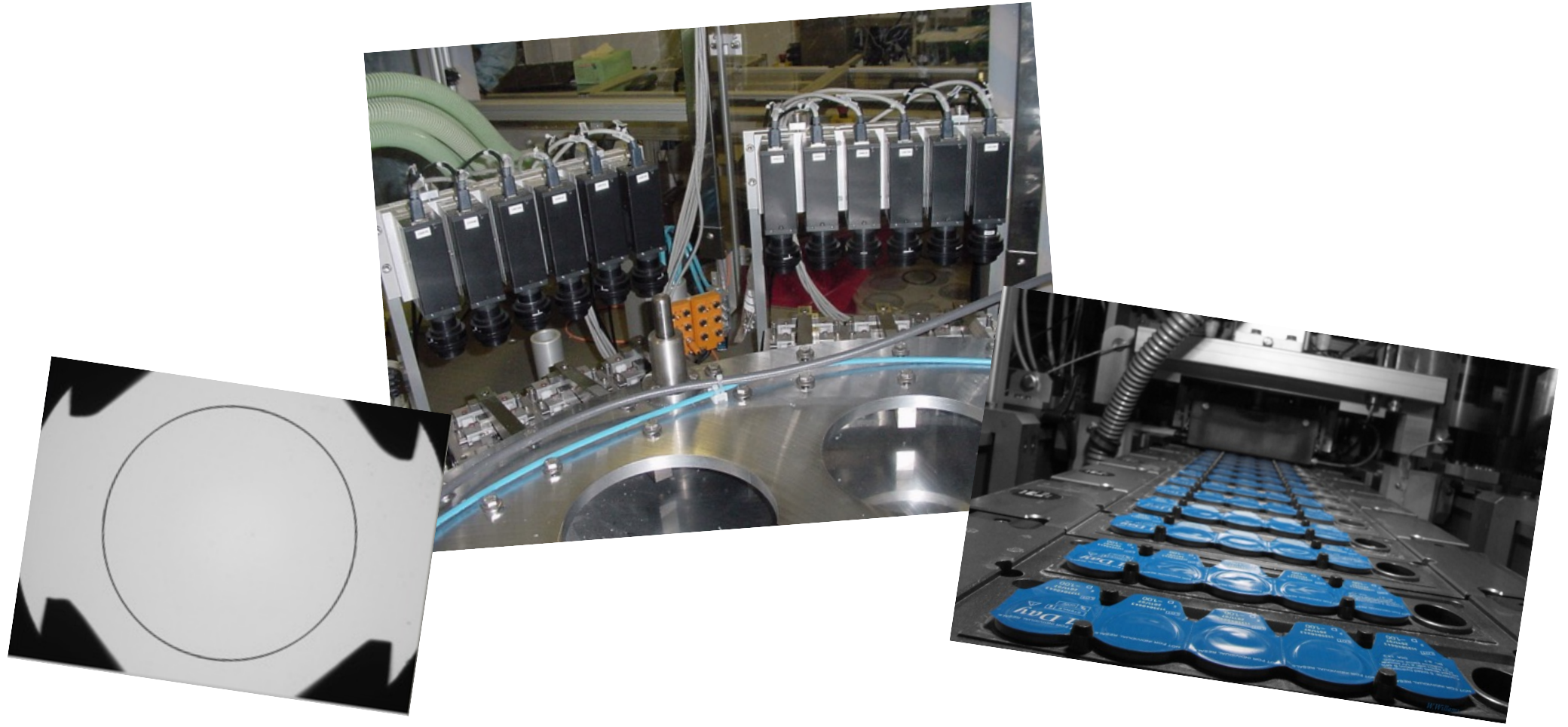


# Integrodifferential operator?



<https://stackoverflow.com/questions/27058057/comparing-irises-images-with-opencv>

# Contact lenses





# Extensions to conic sections

Ellipse

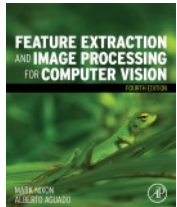
$$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} = 1$$

Described by 4 parameters. If each has **100** values,

$$\text{accumulator size} = 10^2 \times 10^2 \times 10^2 \times 10^2 = 10^8 = 0.1 \text{GB}$$

Add **rotation**, that's 10GB .... Ouch!

Motivates approaches to **save memory** and **improve speed** (since result is optimal)



# Speeding it up.....

Now it's a **3D** accumulator, fast algorithms are available

E.g. by **differentiation**

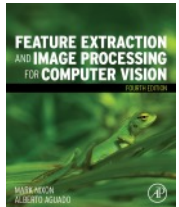
Differentiating  $(x - x_0)^2 + (y - y_0)^2 = r^2$  gives  $\frac{dy}{dx} = -\frac{(x - x_0)}{(y - y_0)}$

Substitute back into Eqn. for circle

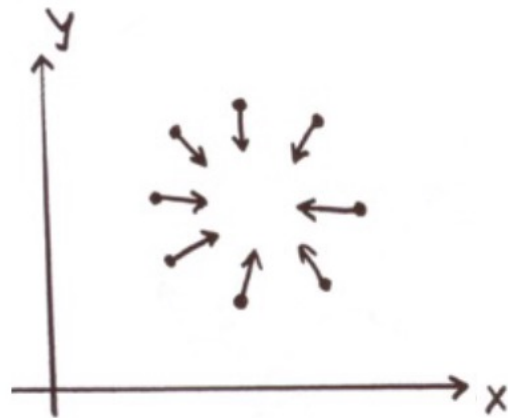
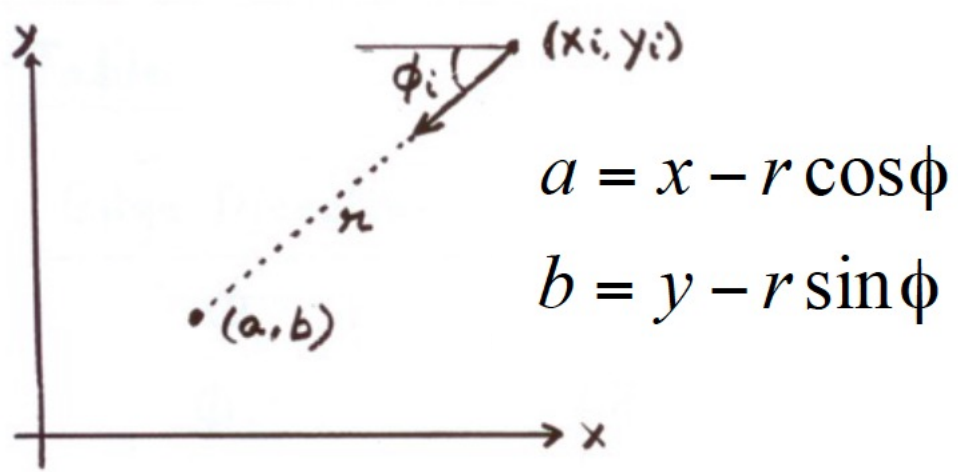
$$\left(\frac{dy}{dx}\right)^2 (y - y_0)^2 + (y - y_0)^2 = r^2 \quad \text{2D accumulator}$$

$$y - y_0 = \frac{r}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}$$

This is the edge direction

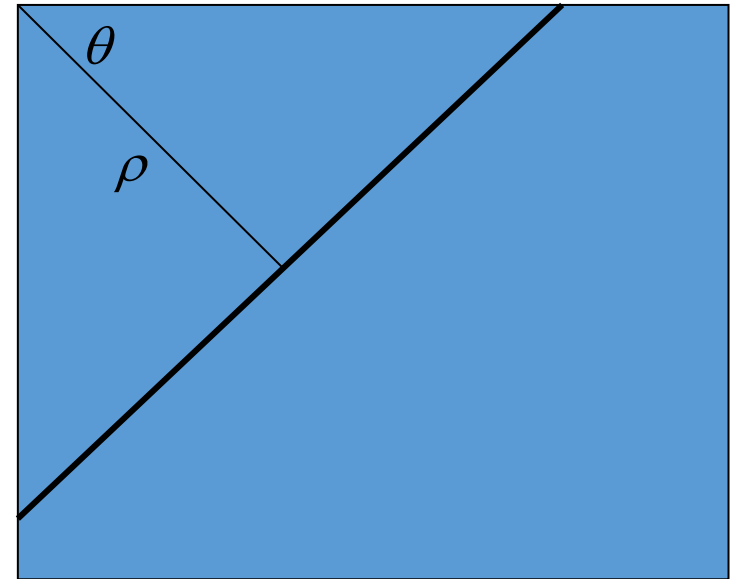


# Fireside



Circle

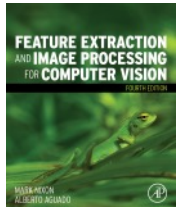
$$\rho = x \cos \theta + y \sin \theta$$



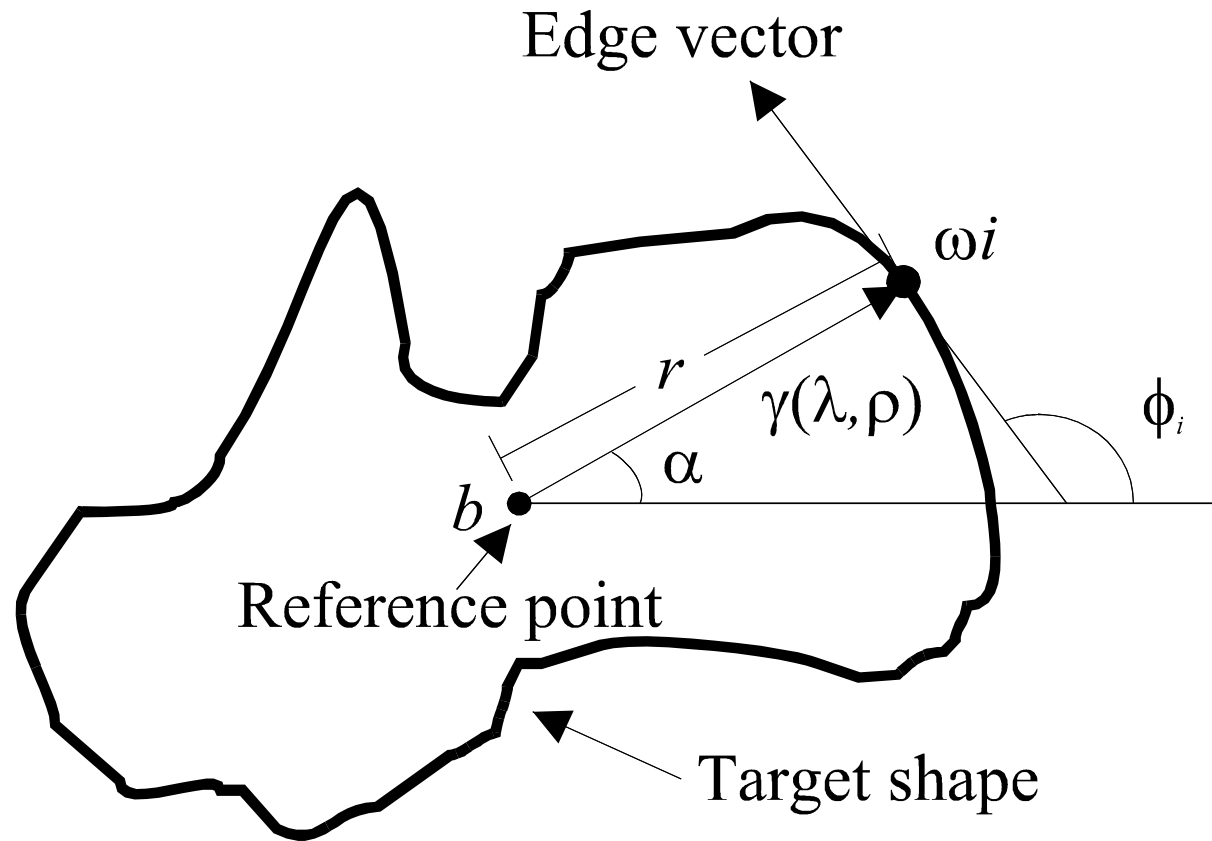
Line

# Arbitrary Shapes

- Use **Generalised** HT
- Form (discrete) **look-up-table** (R-table)
- **Vote** via look-up-table



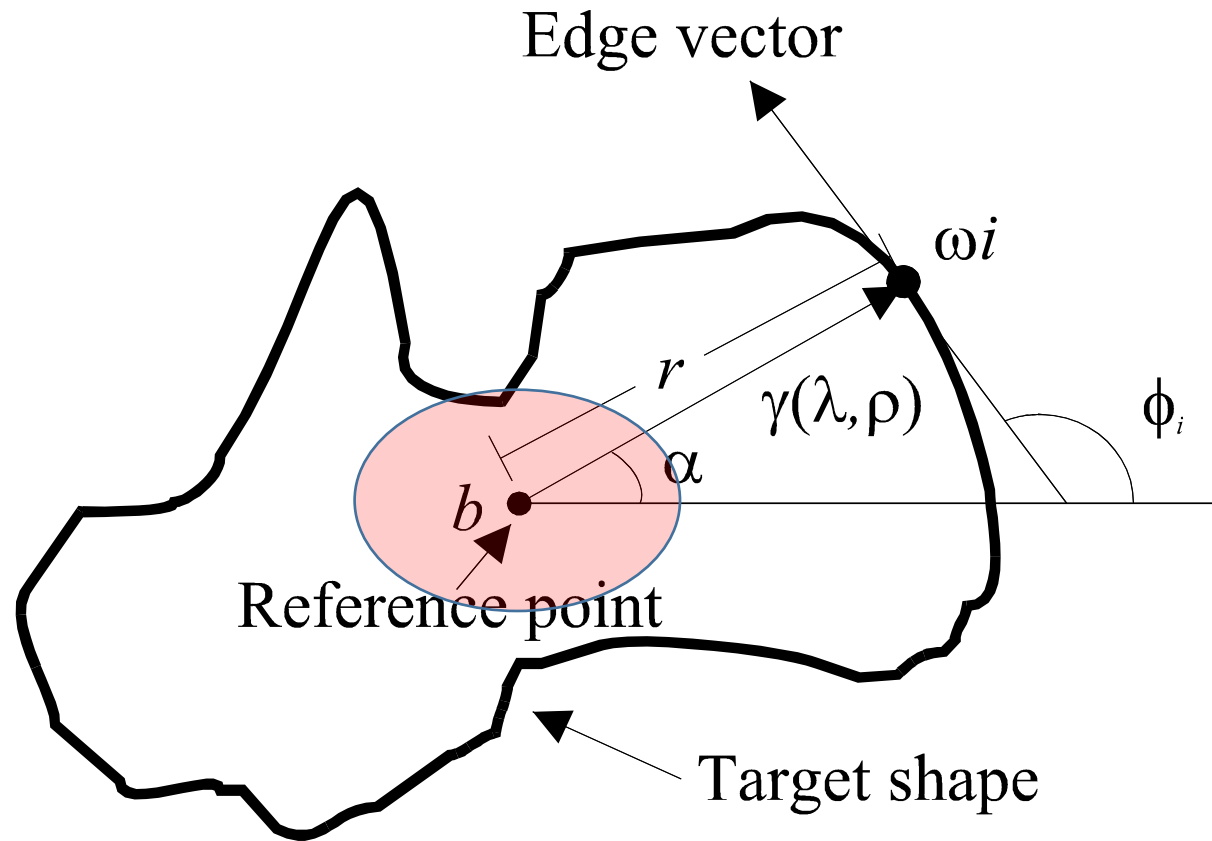
# R-table Construction



$\hat{\phi}_i$	$\gamma = (r, \alpha)$
0	$(r_0, \alpha_0), (r_1, \alpha_1), (r_2, \alpha_2)$
$\Delta\phi$	$\vdots$
$2\Delta\phi$	$\vdots$
...	...



# R-table Construction

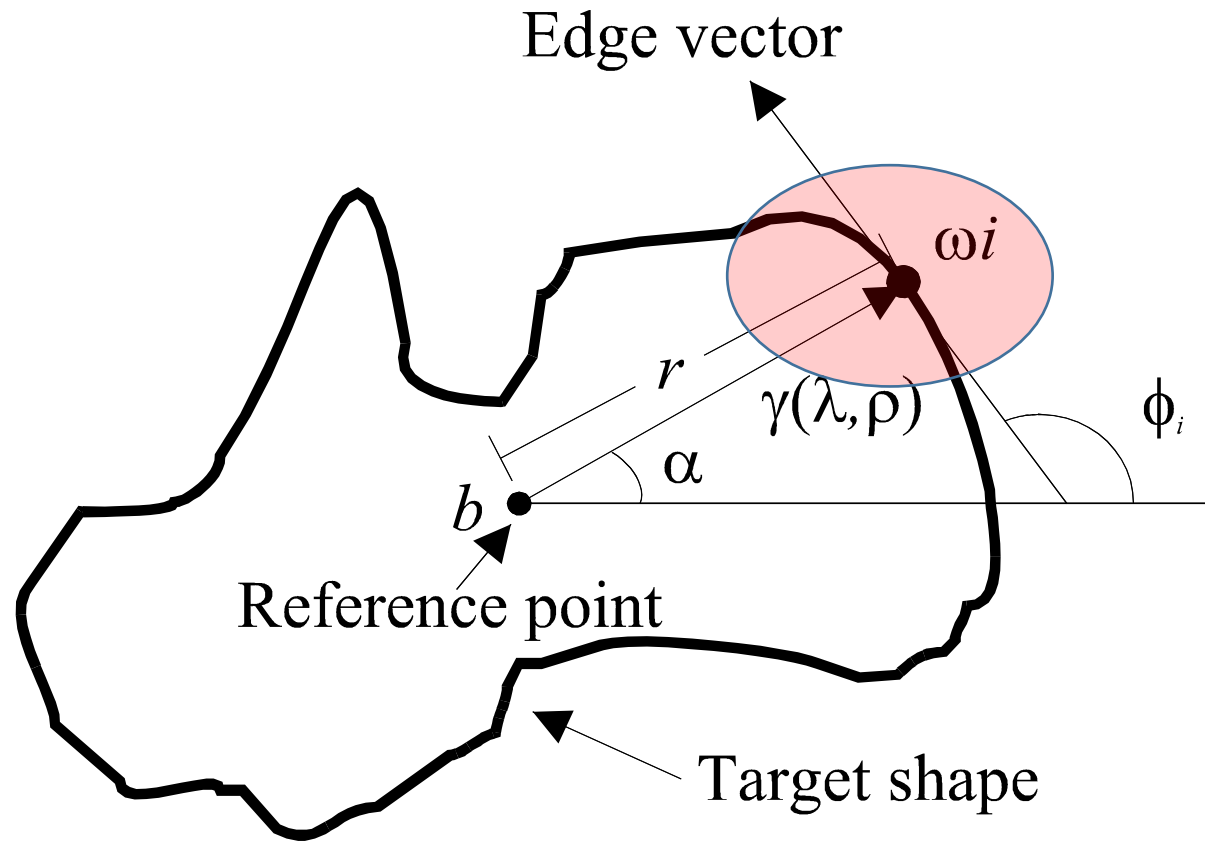


$\hat{\phi}_i$	$\gamma = (r, \alpha)$
0	$(r_0, \alpha_0), (r_1, \alpha_1), (r_2, \alpha_2)$
$\Delta\phi$	$\vdots$
$2\Delta\phi$	$\vdots$
...	...

Need to start somewhere



# R-table Construction

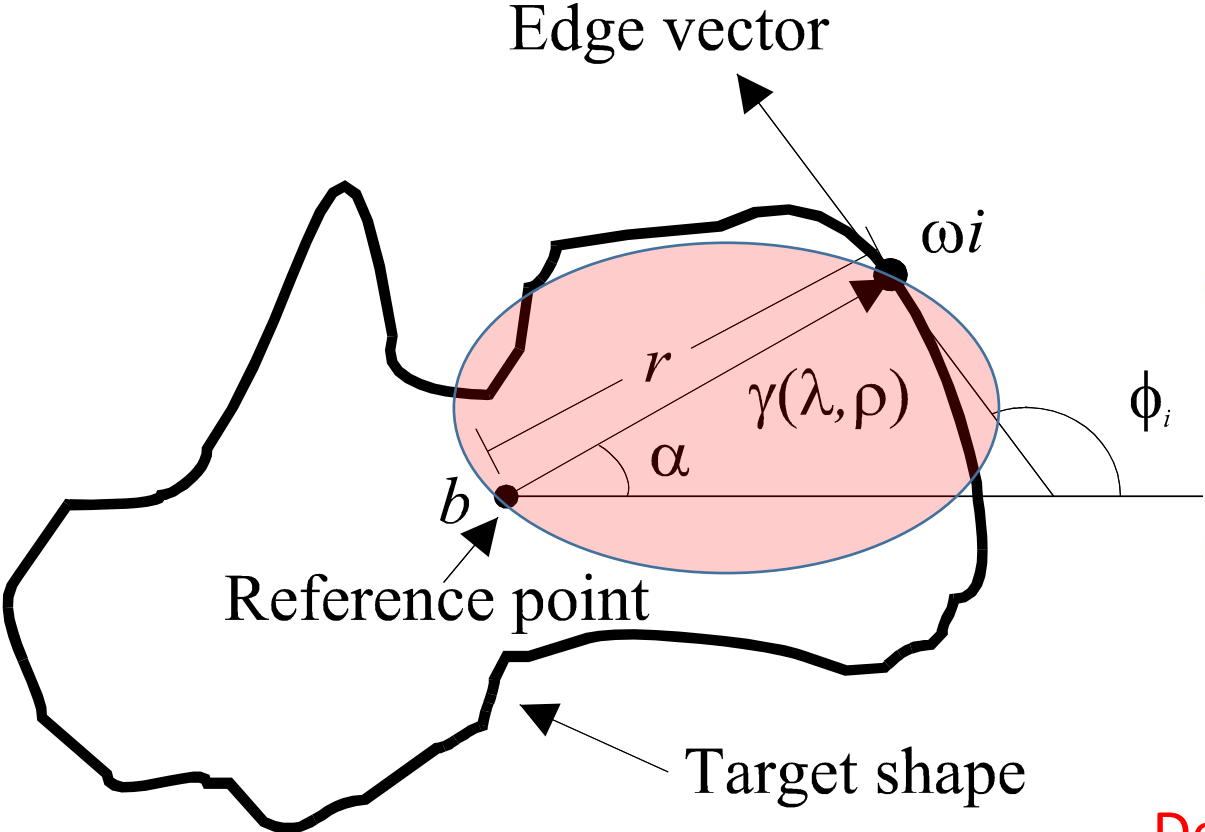


$\hat{\phi}_i$	$\gamma = (r, \alpha)$
0	$(r_0, \alpha_0), (r_1, \alpha_1), (r_2, \alpha_2)$
$\Delta\phi$	$\vdots$
$2\Delta\phi$	$\vdots$
...	...

Measure edge direction



# R-table Construction



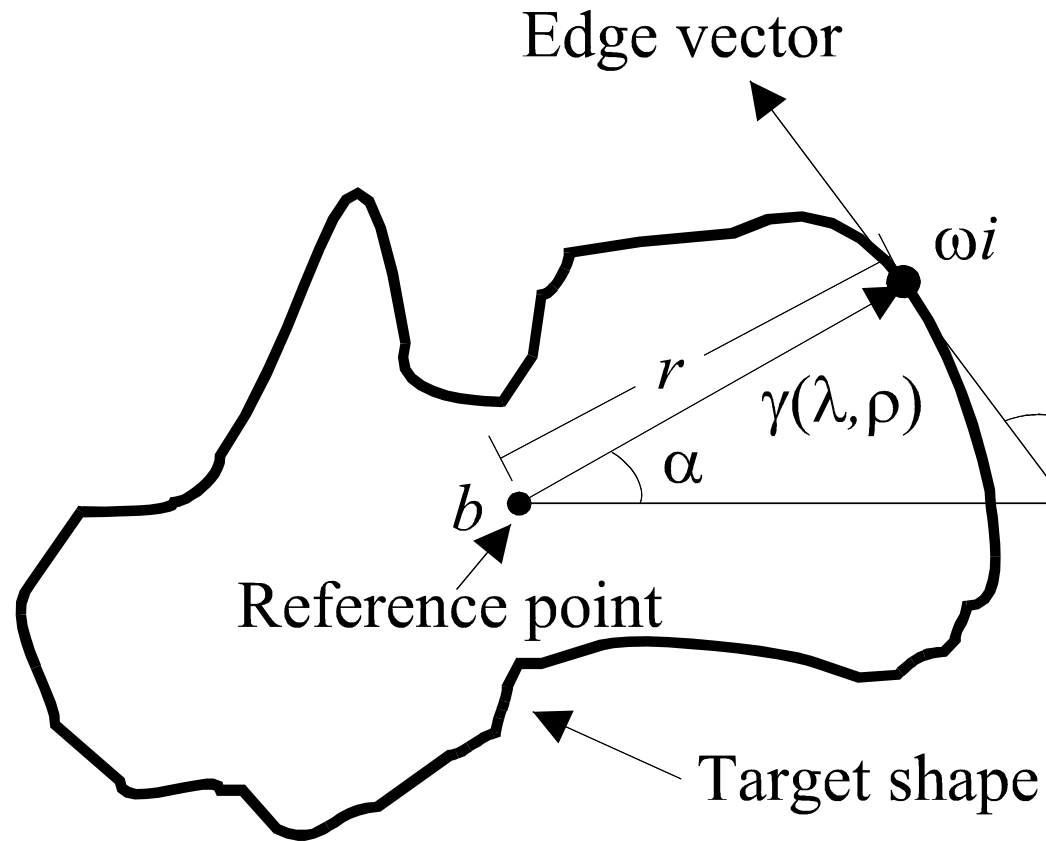
$\hat{\phi}_i$	$\gamma = (r, \alpha)$
0	$(r_0, \alpha_0), (r_1, \alpha_1), (r_2, \alpha_2)$
$\Delta\phi$	$\vdots$
$2\Delta\phi$	$\vdots$
...	...

Determine length and direction to reference point





# R-table Construction



$\hat{\phi}_i$	$\gamma = (r, \alpha)$
0	$(r_0, \alpha_0), (r_1, \alpha_1), (r_2, \alpha_2)$
$\Delta\phi$	$\vdots$
$2\Delta\phi$	$\vdots$
...	...

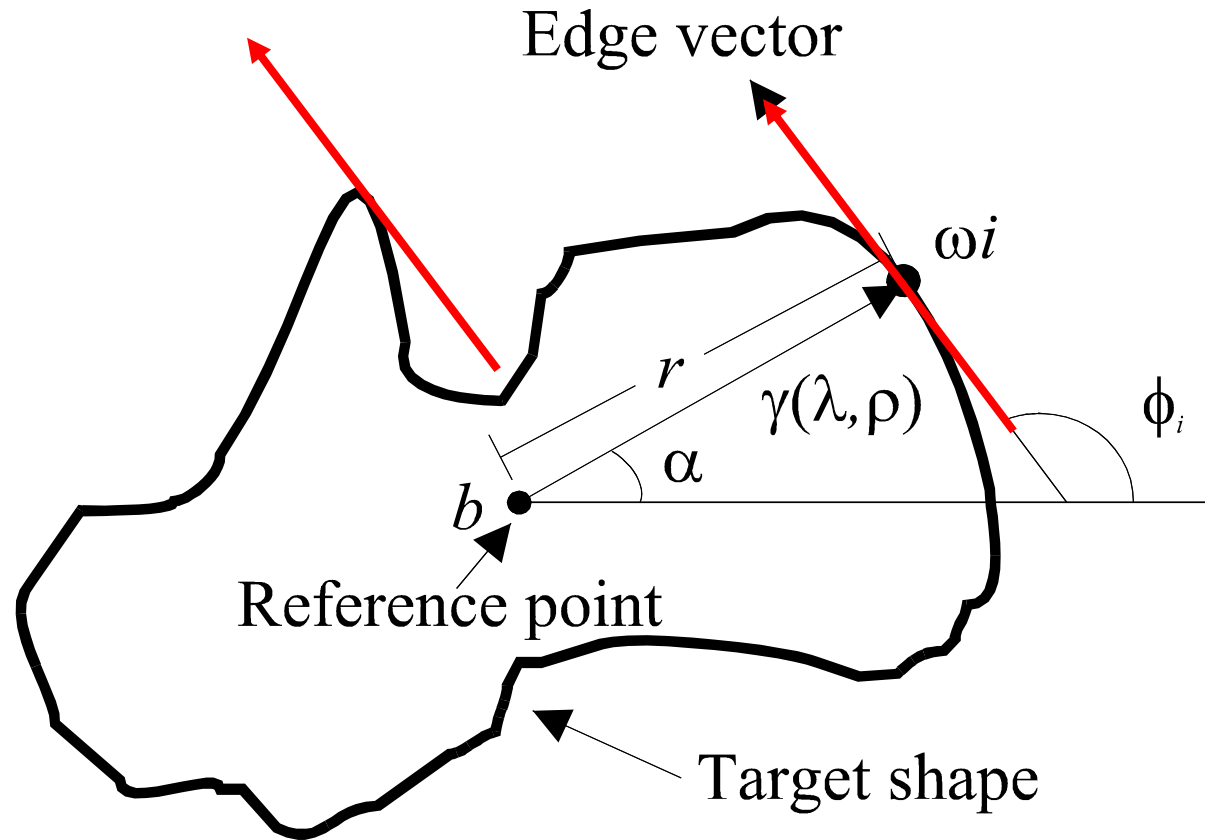
Store length and direction indexed by edge direction



# R-table Construction

$$x_c = x_i - r \cos(\alpha)$$

$$y_c = y_i - r \sin(\alpha)$$



$\hat{\phi}_i$	$\gamma = (r, \alpha)$
0	$(r_0, \alpha_0), (r_1, \alpha_1), (r_2, \alpha_2)$
$\Delta\phi$	$\vdots$
$2\Delta\phi$	$\vdots$
...	...

Edge direction is not a unique description  
Gives noise in accumulator



# Procedure for GHT

## Preparation

1. Determine centre of template shape
2. Form R-table from template shape

## Application

1. Use R-table to vote for points in the real image

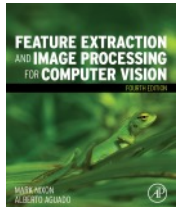
For edge points  $>$  threshold

Get edge direction  $(x, y)$

For all R-table entries with direction  $(x, y)$

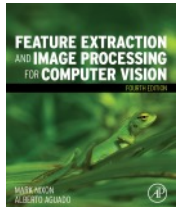
Vote in accumulator  $(@distance, @direction)$

2.  $\text{Argmax}(\text{accumulator})$  gives centre co-ordinates of shape



# Arbitrary Shapes

- Use **Generalised** HT
- Form (discrete) **look-up-table** (R-table)
- **Vote** via look-up-table
- **Scale**? scale R-table voting
- **Orientation**? Rotate R-table voting
- Inherent **problems** with discretisation



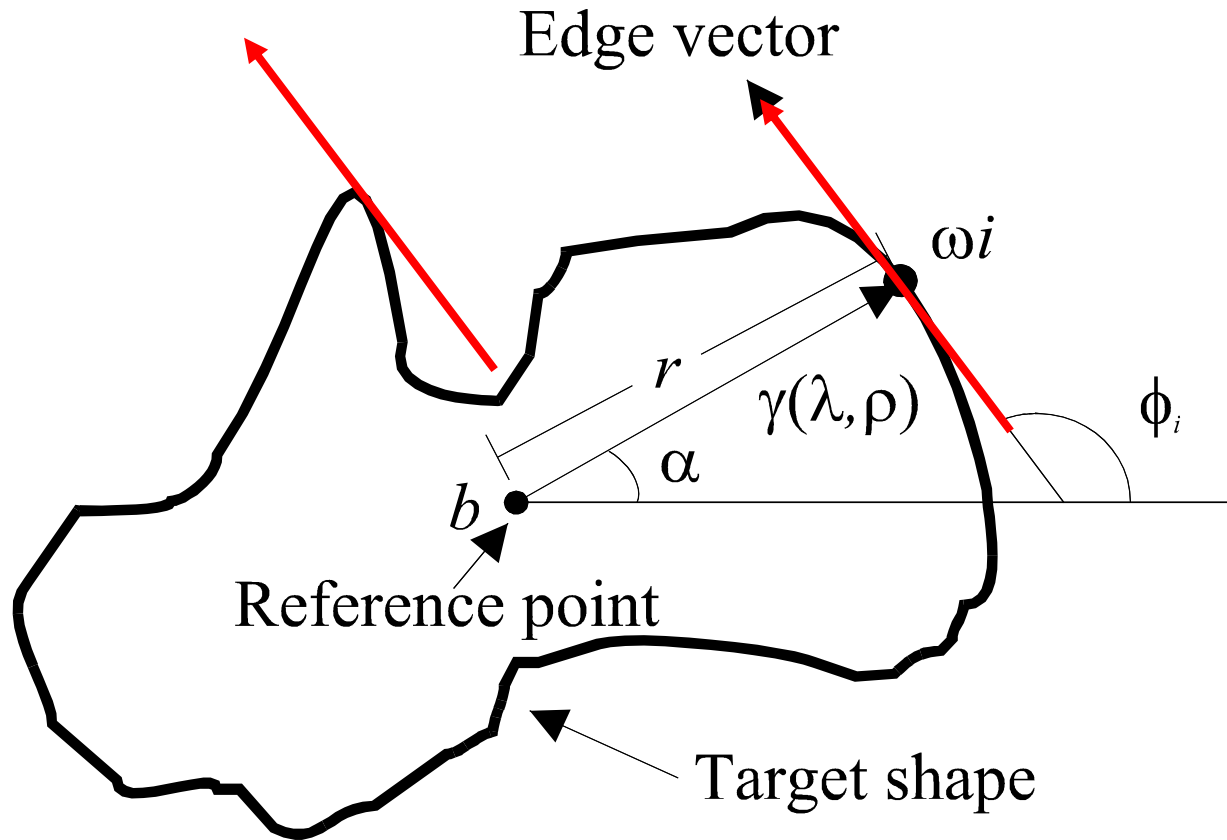
# R-table Construction

$$x_c = x_i - rS \cos(\alpha + \theta)$$

$$y_c = y_i - rS \sin(\alpha + \theta)$$

Scale

Orientation

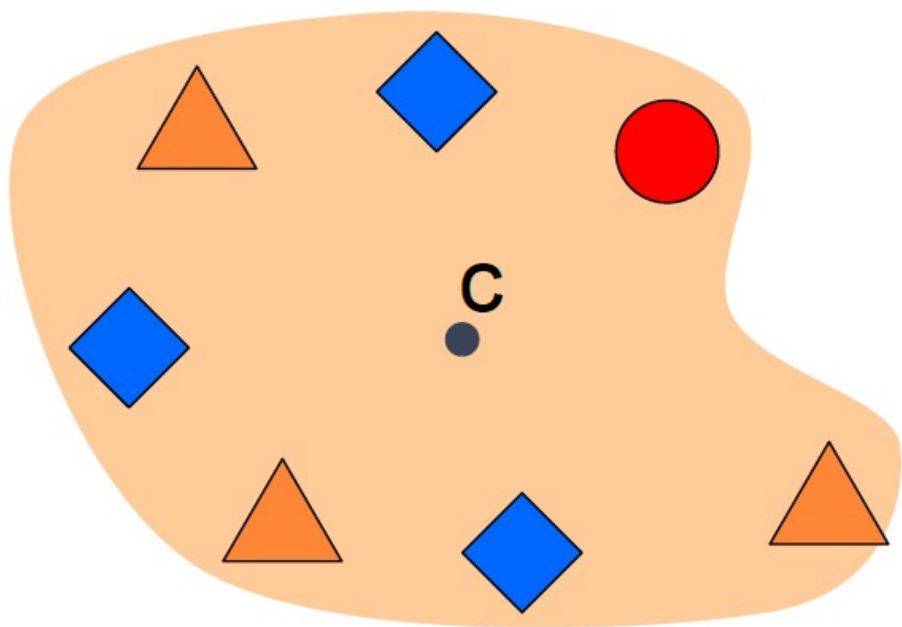


$\hat{\phi}_i$	$\gamma = (r, \alpha)$
0	$(r_0, \alpha_0), (r_1, \alpha_1), (r_2, \alpha_2)$
$\Delta\phi$	$\vdots$
$2\Delta\phi$	$\vdots$
...	...



# Fireside

Template



# Visual inspection



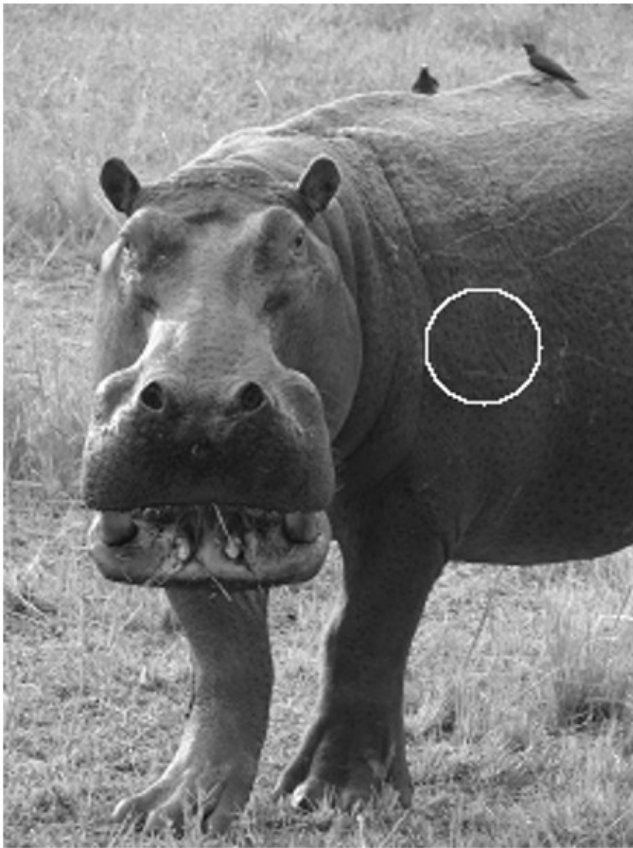
# Active Contours

- For **unknown** arbitrary shapes: extract by **evolution**
- **Elastic band** analogy
- **Balloon** analogy
- Discrete vs. continuous
- **Volcanoes?**





# Geometric active contours



(a) initialisation



(b) result

**Extraction by a Level-Set Based Approach**

# Main points so far

- 1 – **conic sections** become more complex and take more time
- 2 – can use **Generalised Hough Transform** for complex shapes
- 3 – **shape detection** IS computer vision.  
Many more approaches

Let's see how computer vision can work

